Practice Test 1 for Analysis I, Math 4317, September 23, 2010

Always state your reasoning otherwise credit will not be given

1: For any two subsets of $S$ show that
$$A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$$

2: Let $X$ be a finite set and $f : X \to X$ be a function that is one-one. Show that $f$ is onto.

3: Consider the sequence given recursively by $x_0 = 3$ and
$$x_{n+1} = \frac{1}{3}(x_n + x_n^2)$$
Does this sequence converge?

4: Consider the sequence
$$a_n = \frac{2n^2 + 3}{n^2 + n + 1}.$$ Give a rigorous proof that $a_n$ converges to 2.

5: Prove that if a sequence $a_1, a_2, \cdots$ of real numbers converges to some number $a$, then the sequence
$$b_n := \frac{\sum_{k=1}^{n} ka_k}{n^2}$$
also converges. What is the limit of this sequence? Is the converse true?

6: Find a complete metric space and a sequence of bounded closed sets $S_i, i = 1, 2, \cdots$ such that
$$S_1 \supset S_2 \supset S_3 \cdots$$
but
$$\cap_{i=1}^{\infty} S_i = \emptyset$$

7: Prove that every bounded monotone sequence of real numbers converges.

8: Let $A$ and $B$ be subsets of a metric space. Assume that $A$ is closed and $B$ is open. Show that the complement of $A \cap B$ is closed as a subset of the metric space $A$.

9: Find a collection of nonempty closed subsets of the real numbers whose union is bounded and open.

10: Is the set consisting of all rational numbers $r$ with $0 \leq r \leq 1$ a compact subset of the real numbers?