This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414.... Show your work, otherwise credit cannot be given. PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.
I: (25 points) a) Does the series

\[ \sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k} \]

converge absolutely?

b) Consider the series

\[ \sum_{k=0}^{\infty} (-1)^k \frac{2^k}{(2k)!} . \]

Does this series converge? If yes, calculate the first five digits after the decimal point of this limit.

c) Let

\[ \sum_{k} a_k x^k \]

be a power series and assume that it converges at \( c > 0 \). True or false:

1) The series necessarily converges for all \( x < c \).
2) The radius of convergence \( R \) satisfies necessarily \( R \geq c \).
3) The radius of convergence \( R \) satisfies necessarily \( R \leq c \).
4) The series converges absolutely for \( x \) with \( |x| < c \).
II: (25 points) a) Write the power series expansion at $x = 0$ of the function

$$\int_0^x \frac{1}{1 + t^4} dt.$$ 

For which values of $x$ does the series converge? What is the radius of convergence?

b) What is the radius of convergence of the power series

$$\sum_{k=1}^{\infty} (1 + \frac{1}{k})^{k^2} x^k ?$$

c) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{1}{k} (x - 2)^k 2^{-k}.$$ 

$$\sum_{k=2}^{\infty} \frac{\log k}{k^2} x^k$$
III: (25 points) a) Find the intersection the line

\[ \mathbf{x}(t) = \langle 1, 2, 1 \rangle + t\langle 0, 4, 2 \rangle \]

with the plane

\[ x + y + z = 3 \]

b) Find the angle between the planes

\[ 2x - y + 3z = 2 , \ 5x + 5y - z = 4 \]

c) Find the line that forms the intersection of the two planes
IV: (25 points) a) Find the distance of the tip of the vector \( \langle 2, -1, 3 \rangle \) to the plane
\[
2x + 4y - z = -1
\]

b) Find the distance of the tip of the vector \( \langle 1, 2, 3 \rangle \) to the line \( \vec{x}(t) = \langle 1, 0, 2 \rangle + t\langle 1, -2, 3 \rangle \).