This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414.... Show your work, otherwise credit cannot be given.

PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.
Consider the system of equations

\[
\begin{align*}
x - 2y + az &= 2 \\
x + y + z &= 0 \\
3y + z &= 2
\end{align*}
\]

a) (15 points) For which values of a, if any, does this system have a unique solution? Find the solution for any such value of a.

b) (5 points) For which value of a, if any, does this system have infinitely many solutions? Find all the solutions for any such value of a.

c) (5 points) For which value of a, if any, does this system have no solutions?
II: Are the following vectors linearly independent?

\[
\begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix}, \begin{bmatrix}
-2 \\
1 \\
4 \\
-3
\end{bmatrix}, \begin{bmatrix}
-3 \\
-4 \\
1 \\
2
\end{bmatrix}, \begin{bmatrix}
-4 \\
3 \\
1 \\
-2
\end{bmatrix}
\]
III: Do the following vectors span $\mathbb{R}^3$?

\[
\begin{bmatrix}
1 \\
4 \\
2
\end{bmatrix}, \begin{bmatrix}
2 \\
1 \\
3
\end{bmatrix}, \begin{bmatrix}
-1 \\
10 \\
0
\end{bmatrix}
\]
IV: A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ has the following properties

$$T \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \ T \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Find the matrix associated with $T$. 
Consider the linear transformation $T : R^3 \rightarrow R^3$ given by

\[
T(\vec{e}_1) = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad T(\vec{e}_2) = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad T(\vec{e}_3) = \begin{bmatrix} 1 \\ 9 \\ -2 \end{bmatrix}
\]

Is this transformation one-to one?