Practice Test 4A for Calculus II, Math 1502, November 9, 2013

PRINT Name:

PRINT Section:

PRINT Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414.... Show your work, otherwise credit cannot be given. PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.
I: For the matrix

\[
A = \begin{bmatrix}
1 & 2 & 4 & -19 & 7 \\
2 & 5 & 5 & -26 & 9 \\
3 & 6 & 6 & -27 & 9
\end{bmatrix}
\]

a) (10 points) Find a basis for the column space of the matrix \( A \).

b) (10 points) Find a basis for the null space of the matrix \( A \).
II: Consider the subspace $S$ of $\mathbb{R}^3$ spanned by the vectors

\[
\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 12 \end{bmatrix}
\]

a) (10 points) Find a basis for this subspace.

b) (10 points) Now, consider the subspace $T$ of $\mathbb{R}^3$ spanned by the vectors

\[
\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}
\]

How are $T$ and $S$ related?
III: a) (10 points) Compute the inverse of the matrix

\[
A = \begin{bmatrix}
1 & 3 & -1 \\
2 & 2 & 2 \\
3 & 1 & 1 \\
\end{bmatrix}.
\]

b) (10 points) Find the third column of the matrix \(B^{-1}\) where

\[
B = \begin{bmatrix}
-1 & -7 & -3 \\
2 & 15 & 6 \\
1 & 3 & 2 \\
\end{bmatrix}
\]

without computing the other columns.
IV: Compute all the eigenvalues and the corresponding eigenvectors of the matrix

\[
\begin{bmatrix}
6 & -2 \\
6 & -1
\end{bmatrix}
\]

b) (10 points) The matrix

\[
\begin{bmatrix}
2 & 2 & -1 \\
1 & 3 & -1 \\
-1 & -2 & 2
\end{bmatrix}
\]

has the 1 as an eigenvalue. Find the other eigenvalues and the corresponding eigenvectors.
V: (5 points each) Prove or find a counter example. Let $A$ be an $m \times n$ matrix.

a) If the columns are linearly independent then the matrix is invertible.

b) If the columns are linearly independent and span $\mathbb{R}^m$ then $n = m$.

c) If the dimension of the $Nul(A)$ is $n - 1$ then $m = 1$

d) If the dimension of $Col(A) = n$ then $A$ is invertible.