1. Prep-Final B

Problem 1: Find the parametric equations of the line that is tangent to the curve
\[ \vec{r}(t) = (e^t, \sin t, \ln(1 - t)) \]
at \( t = 0 \).

Problem 2: Find the minimum cost area of a rectangular solid with volume 64 cubic inches,
given that the top and sides cost 4 cents per square inch and the bottom costs 7 cents per
square inch. Just set up the equations using Lagrange multipliers, you do not have to solve
them.

Problem 3: Compute the average of the function \( x^4 \) over the sphere centered at the origin
whose radius is \( R > 0 \).

Problem 4: Compute the flux
\[ \int_S \vec{F} \cdot \vec{n} \, d\sigma \]
where \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 4, z \geq 0 \), \( \vec{n} \) points toward the origin and
\[ \vec{F} = (x(z - y), y(x - z), z(y - x)) \, . \]

Problem 5: Compute the line integral \( \int_C \vec{F} \cdot d\vec{r} \) where \( C \) is the curve given by the intersection
of the sphere \( x^2 + y^2 + z^2 = 4 \) and the plane \( z = -y \), counterclockwise when viewed from
above, and
\[ \vec{F} = (x^2 + y, x + y, 4y^2 - z) \, . \]