1. (4.8.49,71,79) Find the indefinite integrals.

(a) (10 points)
\[ \int (7x - 2)^3 \, dx \]

Solution:
(a)
\[ \int (7x - 2)^3 \, dx = \frac{1}{28}(7x - 2)^4 + C. \]

(b) (10 points)
\[ \int \frac{1}{1 + t^2} \, dt \]

Solution:
(b)
\[ \int \frac{1}{1 + t^2} \, dt = \tan^{-1}(t) + C. \]

(c) (5 points)
\[ \int \frac{\cos \theta}{\sin^2 \theta} \, d\theta \]

Solution:
(c)
\[ \int \frac{\cos \theta}{\sin^2 \theta} \, d\theta = -\csc \theta + C. \]
2. (25 points) (5.2.29,35) Find the limit

$$\lim_{n \to \infty} \sum_{j=0}^{n-1} \left[ 16 - \frac{j/n + (j + 1)/n}{6} \right] (x_{j+1} - x_j)$$

where $x_j = j/n$ for $j = 0, 1, 2, \ldots, n$. Hint: Interpret the sum as a Riemann sum with midpoints for the evaluation points.

**Solution:** This is a limit of Riemann sums (with midpoint evaluation points) for

$$\int_0^1 \left[ 16 - \frac{x}{3} \right] \, dx = \left[ 16x - \frac{x^2}{6} \right]_0^1 = 16 - \frac{1}{6} = \frac{95}{6}.$$
3. (5.3.42, 5.6.105)

(a) (10 points) Find the area in the first quadrant under the graph of

\[ f(x) = \frac{9x^{10}}{\sqrt{1 - x^{22}}} \]

and to the left of the line \( x = 1 \).

(b) (15 points) Find the area in the first quadrant bounded below by the line \( y = x/4 \), above left by the curve \( y = 1 + \sqrt{x} \), above right by the curve \( y = 2/\sqrt{x} \), and on the right by the line \( x = 4 \).

Solution:

(a) Make a \( u \)-substitution with \( u = x^{11} \) and \( du = 11x^{10} \, dx \).

\[
A = \int_{0}^{1} \frac{9x^{10}}{\sqrt{1 - x^{22}}} \, dx
\]

\[
= \frac{9}{11} \int_{0}^{1} \frac{1}{\sqrt{1 - u^2}} \, du
\]

\[
= \frac{9}{11} \sin^{-1} u \Bigg|_{0}^{1}
\]

\[
= \frac{9}{11} \sin^{-1} (1)
\]

\[
= \frac{9\pi}{22}.
\]

(b)

\[
A = \int_{0}^{1} \left[ 1 + \sqrt{x} - \frac{x}{4} \right] \, dx + \int_{1}^{4} \left[ \frac{2}{\sqrt{x}} - \frac{x}{4} \right] \, dx
\]

\[
= \left[ x + \frac{2}{3}x^{3/2} - \frac{x^2}{8} \right]_{0}^{1} + \left[ 4\sqrt{x} - \frac{x^2}{8} \right]_{1}^{4}
\]

\[
= 1 + 2/3 - 1/8 + 8 - 2 - [4 - 1/8]
\]

\[
= 11/3.
\]
4. (25 points) (6.1.28) A solid is formed by rotating a certain area in the first quadrant about the $y$-axis. The area is bounded above by the line $y = 2$ and on the right by the graph of $x = e^y$. Find the volume of the solid.

**Solution:** We use the “disks” method:

\[
V = \int_0^2 \pi e^{2y} \, dy \\
= \frac{\pi}{2} \left[ e^4 - 1 \right].
\]