1. (20 points) (6.1.53) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about the line $x = 1$. Hint: Use the method of washers.

Solution: The outer radius of the washer is $1 + \sqrt{z}$ while the inner radius is $1 - \sqrt{z}$. Thus, the volume is

$$V = \int_0^1 \left[ \pi (1 + \sqrt{z})^2 - \pi (1 - \sqrt{z})^2 \right] dz$$

$$= 4\pi \int_0^1 \sqrt{z} \, dz$$

$$= \frac{8\pi}{3}$$
2. (20 points) (6.2.25) Use the method of cylindrical shells to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about the $y$-axis.

**Solution:** The shells are indexed by $x \in [0, 1]$, and the height of the shell at radius $x$ is $x - x^2$. Therefore, the volume by cylindrical shells is

$$V = \int_0^1 2\pi x (x - x^2) \, dx$$

$$= 2\pi \int_0^1 (x^2 - x^3) \, dx$$

$$= 2\pi\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= \frac{\pi}{6}.$$
Solution: We can parameterize this curve by \( x(t) = t \) and \( y(t) = \ln(\cos t) \) for \( 0 \leq t \leq \pi/4 \). The speed of the parameterization is

\[
\sqrt{x'(t)^2 + y'(t)^2} = \sqrt{1 + \tan^2 t} = \sec t.
\]

Thus the length is

\[
L = \int_0^{\pi/4} \sec t \, dt
\]

\[
= \ln(\sec t + \tan t) \bigg|_0^{\pi/4}
\]

\[
= \ln(\sqrt{2} + 1) - \ln(1)
\]

\[
= \ln(1 + \sqrt{2}).
\]
4. (7.1.4,7,19,30) Compute the following integrals
   (a) (5 points)
   \[ \int \frac{8t}{4t^2 - 5} \, dt \]

   (b) (5 points)
   \[ \int \frac{1}{2\sqrt{x} + 2x} \, dx \]

   (c) (5 points)
   \[ \int \frac{e^{1/t}}{t^2} \, dt \]

   (d) (5 points)
   \[ \int_{1}^{4} \frac{2\sqrt{x}}{\sqrt{x}} \, dx \]

Solution:
(a) \( u = 4t^2 - 5 \).
\[ \int \frac{8t}{4t^2 - 5} \, dt = \int \frac{1}{u} \, du = \ln |u| + C = \ln |4t^2 - 5| + C. \]

(b) \( u = 1 + \sqrt{x} \)
\[ \int \frac{1}{2\sqrt{x} + 2x} \, dx = \int \frac{1}{2\sqrt{x}(1 + \sqrt{x})} \, dx = \int \frac{1}{u} \, du = \ln(1 + \sqrt{x}) + C. \]

(c) \( u = 1/t \).
\[ \int \frac{e^{1/t}}{t^2} \, dt = - \int e^u \, du = -e^{1/t} + C. \]

(d) \( u = \sqrt{x} \).
\[ \int_{1}^{4} \frac{2\sqrt{x}}{\sqrt{x}} \, dx = \int_{1}^{2} 2 \cdot 2^u \, dx = 2^{u+1} / \ln 2 \bigg|_{1}^{2} = (8 - 4) / \ln 2 = 4 / \ln 2. \]
5. (20 points) (7.2.3) Solve the initial value problem
\[
y' = -0.6y, \quad y(0) = 100.
\]

**Solution:** \( y(t) = ce^{-0.6t} \) and \( y(0) = 100 \). Therefore,
\[
y(t) = 100e^{-0.6t}.
\]