1. (20 points) (Euler-Cauchy-Wilks ODE; 3.6.10) Solve the ODE

\[ 4x^2y'' + 4xy' - y = 0. \]

2. (20 points) (Second ODE problem; 10.3.4) Use diagonalization to solve the system

\[ x' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} x. \]

3. (20 points) (8.6.8) If the matrix

\[ A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 11 & 14 \\ -1 & 4 & 7 \end{pmatrix} \]

is invertible, use the formula

\[ A^{-1} = \frac{1}{\det A} (A^{\text{cof}})^T \]

to find the inverse. If there is no inverse, explain how you know there is no inverse.

4. (20 points) (8.12.4) Determine if the matrix

\[ A = \begin{pmatrix} 0 & 5 \\ 1 & 0 \end{pmatrix} \]

is diagonalizable. If \( A \) is diagonalizable, find an invertible matrix \( P \) such that \( P^{-1}AP = D \) is diagonal and compute \( D \).

5. (20 points) (17.3.25) Let \( f(z) = z^2 + \bar{z}^2 \) Find

\[ \{ f(z) : \Re z = 2 \text{ and } |z| \leq 4 \}. \]

6. (20 points) (17.5.27) Find the harmonic conjugate of \( u(x, y) = \ln(x^2 + y^2) \).