Spatial Statistical Model for Predicting Crime Behavior Based On the Analysis of Hotspot Mapping

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Abstract
Forced by experimental observations of spatio-temporal clusters of crime across a wide variety of urban settings, we present a model to study the emergence, dynamics, and steady-state properties of crime hotspots. This paper focus on a two-dimensional network model for residential burglary, where each site is illustrated by a dynamic attractiveness variable, and where each criminal is represented as a hit and miss walker. The dynamics of criminals and of the attractiveness field are coupled to each other via specific unfairness and feedback mechanisms. Depending on parameter choices, this paper observe and describe several regimes of aggregation, including hotspots of high criminal activity. On the basis of the discrete model, we also derive a continuum model; the two are in good quantitative agreement for large system sizes. By means of a linear stability analysis we are able to determine the parameter values that will lead to the creation of stable hotspots. This paper discuss our model and results in the context of established criminological findings of criminal behavior.

Keywords: Crime models; reaction-diffusion equations; linear stability.

1. Introduction
One unfortunate aspect of current life is the presence of crime in every major urban area. However, while crime itself is everywhere, it does not appear to be uniformly distributed within space and time. For example, while some neighborhoods tend to be reasonably safe, others appear far more dangerous and display dense clusters of both property and violent crimes.[1],[25–27] These spatio-temporal aggregates of criminal occurrences are commonly referred to as crime “hotspots,” and thanks to recent advances in mapping technology it is possible to track their evolution at fine spatial and temporal scales.[44] The typical lifetimes and length scales of crime hotspots are observed to vary depending upon the particular geographic, economic, or seasonal conditions present. Also, depending on the specific category of crime in question, hotspots are seen to emerge, diffuse and dissipate in ways suggestive of a structured, albeit complex, underlying dynamics.

Many theories have been presented within the criminology community to understand why hotspots emerge in some locations rather than others, how they evolve, and how their “macroscopic”
size and lifetime features are connected to the “microscopic” behaviors of offenders, victims, law enforcement agents, and the local geography.

How these frequent micro scale actions and environmental variables combine to generate higher scale crime patterns is still a matter of debate. This is true even for the relatively simple case of residential burglary where the spatial distribution of targets (i.e. houses) remains constant over time (Fig. 1). What drives the emergence of different burglary patterns must be related not only to how offenders move within their environments, but also to how they respond to the successes and failures of their illicit activities. For example, residential burglars prefer to return to a previously burglarized house, or the ones adjacent to it, in part because it is at precisely these locations where they have good information about the types of property that might be stolen and the schedules of residents.[25],[46] These are known as repeat or near-repeat events, depending upon whether the burglar revisits the same home or one of its neighbors, respectively.

Figure 1: Dynamic changes in residential burglary hotspots for two consecutive three-month periods beginning June 2008 in Promoter Apartments, INDIA. These density maps were created using ArcGIS.

The goal of this paper is to present a quantitative mathematical model that captures the essential dynamics of hotspot formation in light of the above sociological observations. We shall focus on residential burglary, which in many ways is the simplest crime type, since mobile offenders are coupled to stationary target sites, and further complexity arising from the relative movement between the agents at play may be ignored. Our starting point is a discrete network system where every site corresponds to a target house. The network is further characterized by a series of offender agents moving from site to site according to specific rules. As we shall better illustrate in Sec. 2, burglar dynamics are strongly coupled to the level of attractiveness of target sites, with offender movement and rate of burglary biased towards more desirable locations. This unfairness could arise due to the fact that certain homes may indeed be easier to break into, or that these houses might simply be seeming to be better targets. The criminological effects described earlier will be incorporated into our model by letting the degree of attractiveness of each site be a dynamic, non-uniform quantity dependent upon both previous burglary events at the same location and memory effects from burglaries at neighboring sites. We will be interested in the role of this feedback loop on the dynamics and morphology of the criminal hotspots. A continuum derivation based upon the discrete model will also be presented. Here, we common little bit our discrete grid so that burglars are locally described by a number density function, and interactions with the environment are embodied via coupling of this function with the common little bit attractiveness. Our scale crime model will consist of two coupled reaction-diffusion-like equations describing the spatio-temporal evolution of number density and attractiveness, giving rise to hotspot formation.
2. Discrete Model
2.1. Overview

Our discrete burglary model consists of two components, the houses at which burglaries occur, and the criminal agents that commit these burglaries. The houses are imagined as existing on a two-dimensional lattice; for simplicity, we choose a rectangular grid with constant lattice spacing $\ell$ and periodic boundary conditions, though more complicated arrangements that better reflect the layout of an actual apartment complex are possible. In conjunction with the lattice spacing $\ell$, a discrete time unit $\delta t$ over which criminal actions will occur is also chosen. Each house is described by its lattice site $s = (i, j)$ and a quantity $A_s(t)$, which we will refer to as the attractiveness of the site. As the name implies, $A_s(t)$ is a measure of the burglars’ perception of the attractiveness of the home at site $s$, and we model it as being equivalent to the statistical rate of burglary at site $s$ when a burglar is present. We make no attempt to derive this attractiveness from underlying properties of the residence, such as value, security, or location. Instead, we treat the attractiveness in the spirit of collective behavior, modeling it after the sociological phenomena of repeat and near-repeat victimization and the broken windows effect discussed in the introduction. With this in mind, we let

$$A_s(t) = A^0_s + B_s(t), \hspace{1cm} (2.1)$$

where $A^0_s$ represents a static, though possibly spatially varying, component of the attractiveness, and $B_s(t)$ represents the dynamic component associated with repeat and near-repeat victimization.

The criminal agents in our model may perform one of two actions during any given simulated time interval: burglarize the house at which they are currently located, or move to one of the neighboring houses. Burglary is a random event that is characterized by a probability of occurrence for each burglar located at site $s$ between times $t$ and $t + \delta t$ given by

$$p_s(t) = 1 - e^{-A_s(t)\delta t}. \hspace{1cm} (2.2)$$

This probability is in accordance with a standard Poisson process in which the expected number of events during the time interval of length $\delta t$ is $A_s(t)\delta t$. Whenever the site $s$ is burglarized, the corresponding criminal agent is removed from the lattice at that time. This removal represents the tendency of actual burglars to escape the location of their crime after committing it. Burglars are here assumed to simply return home with their burglary goods and to abstain from further crime for the time being. To simulate the removed burglars returning to active status, burglars are also generated at each lattice site at a rate $\Gamma$. This rate could in principle be spatially varying, though we will consider only the case of a uniform value.

If the criminal agent chooses not to burglarize its current location, it will then move to one of the neighboring spots on the grid. This movement will be treated as a random walk process that is biased toward areas of high attractiveness; the justification for this choice is threefold.

1) It is well known that criminals predominantly search for and victimize individuals or property in very local areas surrounding the locations that they routinely visit such as home, work, or places of recreation.\textsuperscript{10}

2) Journey-to-crime distributions generally show that the distances that criminals are willing to travel away from their primary residence to connect in crime is a monotonically decreasing function.\textsuperscript{36}

3) In the case of residential burglary, the tendency to stay close to home often outweighs gains that might be had in traveling farther to victimize more desirable targets.\textsuperscript{5,6}

Random walk models should therefore be appropriate for studying how criminal offenders encounter criminal opportunities, because the behavior of these models is fundamentally local.

We generate the abovementioned criminal motion in our model by defining the probability of movement from site $s$ to the neighboring site $n$ as

$$q_{s \rightarrow n}(t) = \frac{A_s(t)}{\sum_{s' \neq s} A_{s'}(t)} \hspace{1cm} (2.3)$$
where the notation $s' \sim s$ indicates all of the sites neighboring site $s$. Note that, by enforcing that a criminal agent will move exactly one grid-spacing $\ell$ within any time step $\delta t$, we have essentially defined the movement speed of the criminals, and must choose our grid spacing $\ell$ and time interval $\delta t$ in accordance with each other so that this speed adopts a reasonable value.

In the case of residential burglary, it has been suggested that individual residences experience an elevated risk of being re-victimized in a short period of time after a first break in.[24],[25] We introduce such repeat victimization by letting the dynamic attractiveness $B_s(t)$ depend upon previous burglary events at site $s$. Specifically, every time a house is burglarized, we increase $B_s(t)$ for that site by a quantity $\theta$, so that the probability for subsequent burglary events at that home increases via Eq. (2.2). It is reasonable to suppose, however, that this increased probability of burglary at a house has a finite lifetime, and as time progresses the attractiveness returns to the baseline value. We model this increase and decay according to the update rule

$$B_s(t + \delta t) = B_s(t)(1 - \omega \delta t) + \theta E_s(t),$$

(2.4)

where $\omega$ sets a time scale over which repeat victimizations are most likely to occur and $E_s(t)$ is the number of burglary events that occurred at site $s$ during the time interval beginning at time $t$.

Finally, we model near-repeat victimization[24],[25] and the broken windows effect by allowing $B_s(t)$ to spread spatially from each house to its neighbors. This is accomplished by modifying Eq. (2.4) to read

$$B_s(t + \delta t) =\left(1 - \eta \theta \frac{\Delta B_s(t)}{Z}\right)B_s(t)(1 - \omega \delta t) + \theta E_s(t),$$

(2.5)

where $z$, the coordination number, is the number of sites $s'$ which neighbor $s$ (four for the square lattice), and $\eta$ is simply a parameter between zero and unity that measures the significance of neighborhood flows. Higher values of $\eta$ lead to a greater degree of spreading of the attractiveness generated by any given burglary event, and lower values lead to the opposite. Equation (2.5) can be rewritten in the form

$$B_s(t + \delta t) = [B_s(t) + \frac{\eta \theta^2}{Z}\Delta B_s(t)](1 - \omega \delta t) + \theta E_s(t)$$

(2.6)

$$\Delta B_s(t) = \sum_{s' \sim s} \frac{B_{s'}(t) - \theta B_s(t)}{\ell^2}$$

(2.7)

Figure 2: Flow chart for the computer simulations
where $\Delta$ is the discrete spatial Laplacian operator.

Figure 2 presents a visual summary of this section of the paper in the form of a flowchart.

The simplest case for our discrete system is the spatially homogeneous equilibrium solution. Here, all sites have the same attractiveness $A$, and, on average, the same number of criminals $n$. For the attractiveness of any given site to stay fixed, the amount by which the attractiveness decays in one time step must be equal to the amount by which it increases due to burglary events:

$$\omega B \delta t = \theta n \bar{p}$$

Similarly, in order for the number of criminals at a site to remain fixed, the number of criminals removed in one timestep (equal to the number of burglary events during that timestep) must be equal to the number of criminals produced at that site at rate $\Gamma$:

$$\bar{n} \bar{p} = \Gamma \delta t$$

Table 1: Summary of parameters present in the discrete model.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell$</td>
<td>Grid spacing</td>
</tr>
<tr>
<td>$\delta t$</td>
<td>Time step</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Dynamic attractiveness decay rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Measures neighborhood effect ranging from 0 to 1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Increase in attractiveness due to one burglary event</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Intrinsic attractiveness of site</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Rate of burglar generation at each site</td>
</tr>
</tbody>
</table>

2.2. Computer Simulations

Computer simulations of the model described above follow the general sketch as shown in Fig. 2. The main purpose of the simulations is to give insight into the behavior of the model under various combinations of the many parameters present (see Table 1).

By varying these parameters, we observe three distinct behavioral rules for the attractiveness field $A_s(t)$:

1) Spatial homogeneity. In this rule, the attractiveness field has essentially the same value at all points. Any local increases in the field due to recent burglaries disappear very quickly.

2) Dynamic hotspots. In this rule, localized spots of increased attractiveness form and remain for varying lengths of time. These spots may remain mostly fixed in space during their lifetime, or they may appear and disappear at seemingly random locations. Also, the degree of disparity in attractiveness between those areas within the hotspots and not within the hotspots depends upon the parameter choices.

3) Stationary hotspots. In this rule, the system tends toward a steady state in which stationary spots of high attractiveness are found, surrounded by areas of extremely low attractiveness. The size of these spots varies depending upon the parameters chosen.

Some example output from the simulation for each of the cases above can be seen in Fig. 3, where we display color-maps of the attractiveness field as it progresses in time for various sets of parameters. The spatially standardized balance value of the dynamic attractiveness $B_s$ serves as a midpoint, and is shaded in green. Other values of attractiveness follow the rainbow spectrum from violet, corresponding to $B_s = 0$, to red, corresponding to $B_s \geq 2 \bar{B}$. For these particular simulations, parameters were chosen to represent possibly realistic values for those quantities which lend themselves well to estimation.

**Figure 3:** Output from the discrete simulation, using parameters described in the text. In low criminal numbers (b) and (d), observe dynamic hotspots. Those in (b) are more passing in nature, while those in (d) linger but display large deformations over time. For higher criminal numbers, we observe either (a) no major hotspots, or (c) inactive hotspots.

All four were run with $\ell = 1$, $\delta t = 1/100$, $\omega = 1/18$, and $A^0 = 1/40$, where time may be interpreted in units of days, and distance in units of house separation. In this case, the difference between the three rules of behavior arises by varying $\eta$, $\theta$, and $\Gamma$: in Fig. 3(a), $\eta = 0.2$, $\theta = 0.56$ and $\Gamma = 0.019$; in Fig. 3(b), $\eta = 0.2$, $\theta = 5.5$, and $\Gamma = 0.003$; in Fig. 3(c), $\eta = 0.03$, $\theta = 0.55$ and $\Gamma = 0.018$; and in Fig. 3(d), $\eta = 0.03$, $\theta = 5.5$, and $\Gamma = 0.003$. All simulations were performed on a $130 \times 130$ grid, with initial conditions $B_s(0) = \overline{B}$, and the number of criminals at each site $n_s(0)$ being, on average, equal to $\overline{n}$.

We observe that the difference between those systems that exhibit behavior (2) (dynamic hotspots) and those that exhibit behavior (1) (no hotspots) and (3) (stationary hotspots) lies essentially in the relative amount of stochasticity present for the parameters chosen. Those simulations that exhibit large numbers of criminals or burglary events are more likely to fall into rules (3) or (1) than (2), while those with low criminal numbers or low numbers of events behave in the opposite way. This seems to suggest two things: that rules (1) and (3) are indeed two different phenomena, and that rule (2) is really only a different demonstration of either (1) or (3) arising due to finite size effects. In an effort to gain a better understanding of this, we now turn to the derivation of a range approximation of our discrete model.
3. Range Limit
3.1. Derivation

Let us begin the derivation of our range limit by analyzing the dynamics of $B_s(t)$ in greater detail. We can, as a first step, express the expected value of the dynamic attractiveness after one timestep as

$$B_s(t+\delta t) = (B_s(t) + \frac{\eta}{Z} \Delta B_s(t)) (1 - \omega \delta t) + \theta_s(t) p_s(t).$$  \hspace{1cm} (3.1)

We now convert $n_s(t)$ into a number density by simply dividing by $\ell^2$, and renaming it $\rho(x, t)$. We subtract $B_s(t)$ from both sides of the equation and then divide the equation by $\delta t$. Finally, we take the limit as both $\delta t$ and $\ell^2$ become small with respect to the spatial and temporal scales of interest, with the constraints that the ratio $\ell^2/\delta t$ remain fixed with a value we define as $D$, and that the quantity $\theta \delta t$ also remain fixed with a value $\epsilon$. The resulting equation gives the dynamics of the range version of the attractiveness,

$$\frac{\partial B}{\partial t} = \frac{\eta D}{Z} \nabla^2 B - \alpha B + \epsilon D \rho A$$  \hspace{1cm} (3.2)

The derivation of the range limit for $n_s(t)$ is slightly more involved. We begin with an equation expressing the expected number of agents at a site after one timestep, noting that our model demands that all of the agents that were at the site $s$ at time $t$ must have left the site either by moving to a neighboring site or by burglarizing the site and thereby being removed. Because of this, any agents that are present after one timestep must have either arrived there from a neighboring site after failing to burglarize the neighbor or have been generated there at rate $\Gamma$. Therefore, we conclude that

$$n_s(t+\delta t) = A_s \left[ n_s'(t) \left[ 1 - \frac{p_s'(t)}{\rho_s(t)} \right] \right]$$  \hspace{1cm} (3.3)

where, for sake of notational simplicity, we have defined

$$T_s'(t) \equiv \sum_{s' \sim s} A_{s'}(t)$$  \hspace{1cm} (3.4)

Now, we perform an operation like that done previously when converting from Eq. (2.5) to (2.6) to write the sum in Eq. (3.3) and $T_s(t)$ in terms of the discrete spatial Laplacian. We then subtract $n_s(t)$ from both sides of the equation, re-express $n_s(t)$ in terms of $\rho(x, t)$, and divide by $\delta t$. Upon taking the limits of $\ell$ and $\delta t$ as described previously, with the further constraint that $\Gamma/\ell^2 = \gamma$, we arrive at our range equation for criminal number density

$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho + \frac{2p}{A} \nabla A \rho - p \rho A + \gamma.$$  \hspace{1cm} (3.5)

Equations (3.2) and (3.5) are the main results of our range derivation, and are of the general form of a reaction-diffusion system; such systems often lead to pattern formation. The attractiveness diffuses throughout the environment while simultaneously decaying in time and reacting with the criminals to create even more attractiveness. Criminals are depleted through reactions with the attractiveness and are created at a constant rate. In addition, the criminals exhibit both diffusive motion and adjective motion up gradients of attractiveness, with a speed that is inversely proportional to the local attractiveness field. This can be interpreted in a sociological sense as an example of diminishing returns; if an o ender is already located at a highly attractive home, it may feel less motivation to move to neighboring houses that are, relatively speaking, not that much more attractive.
Figure 4: Output from the range simulation. The range parameters used in (a) are the equivalent of the discrete parameters used in both Figs. 3(a) and 3(b), and we observe no hotspots forming. The range parameters used in (b) are the equivalent of the discrete parameters used in both Figs. 3(c) and 3(d), and we observe stationary hotspots with roughly the same size as those seen in Fig. 3(c). Finally, we illustrate stationary hotspots of a different size in (c).

In Figs. 4(a) and 4(b), we have used continuum parameters that are the equivalent of those used to create the plots in Fig. 3; Fig. 4(c) illustrates hotspots of a different size, using the same parameters as in Fig. 4(b) but with $\eta = 0.05$. All three are run on a 512 x 512 network with initial conditions at consistent stability, with the exception of a few numerical grid points that are with a slightly higher $B$ value.

4. Conclusions

We re-emphasize at this point that the model described in this paper is constructed based upon the empirically known behavior of criminal offenders. First, based on the fact that burglars most often victimize areas near where they live, work, or spend free time, we have chosen to model their movement as a biased random walk, as the behavior of such a model is fundamentally local in space. Second, as it is clear that repeat victimization plays an important role in crime pattern generation, we have developed the idea of an attractiveness field that not only determines the rate of burglary at a given site, but is also influenced by past burglary events and serves as the source of bias in the criminals’ movement. Finally, we have introduced spatio-temporal scales for hotspots by allowing our attractiveness field to diffuse within a neighborhood while simultaneously decaying in time. We thus are able to construct a model where the two main variables at play, offender position (or density in the continuum model), and biasing attractiveness field, create nonlinear feedback loops which originate patterns of aggregation, reminiscent of actual crime hotspots.

This sociologically based model accomplishes our chief goal of exhibiting qualitative similarity with the hotspots observed in actual cities. However, there has been no comparison as of yet between the quantitative aspects of the hotspots generated thereby and empirical crime data. This is partly because of the difficulty in developing a rigorous metric by which such a comparison could be made. To wit, there are numerous quantities that can be measured in both our simulation output and empirical burglary data that could serve as such a rubric: the probability distribution for number of burglaries per house over a prescribed period of time, the distribution of time to next event for houses within a fixed distance of a burglary event, any number of tests for spatiotemporal clustering of burglary events, etc. Choosing which one of these measures to focus our attention toward is a work in progress. In the end, this knowledge may eventually prove useful for developing better methods of crime prediction and prevention and allow the police and other security agencies to more effectively control resource allocation from day to day.

References


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