Worksheet 4: Chapter 2/3 (ε-δ defn, limits of the secant/DQ)

1. From the definition of
\[ \lim_{x \to 3} x^2 = 9 \]
Find the largest \( \delta > 0 \) such that if \( |x - 3| < \delta \), then \( |x^2 - 9| < \varepsilon \), where \( \varepsilon = 2 \).

2. Let \( f(x) = \sqrt{x} \). Find
\[ L = \lim_{x \to 16} f(x). \]
Then, find the largest \( \delta > 0 \) such that
\[ |x - 16| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon \]
for \( \varepsilon = 4 \).
3. Consider the function \( f(x) = \frac{1}{x} \), and we will examine a secant line of \( f(x) \) as well as \( f'(2) \) which is the derivative of \( f \) at \( x = 2 \), the limit of the slopes of the secant lines near \( x = 2 \):

\[
f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.
\]

(a) Find the slope of the secant line of the points \((2, 1/2)\) and \((4, 1/4)\).

(b) Guess the limit of the slopes of the secant lines at \( x = 2 \) by either creating a table (in a spreadsheet) or using graphing software (like GeoGebra).

(c) Find the instantaneous rate of change at \( x = 2 \). That is, find

\[
\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.
\]

Note that “instantaneous rate of change” and “limit of the slopes of the secant lines” and “the derivative” and “limit of the difference quotient” are all (essentially) synonyms.
4. On a walk with my dog my distance from home after $t$ minutes is given by

$$f(t) = \frac{t^3}{8} - t^2 + 2t.$$

What is my average speed in the first 30 seconds of my walk? (1) What is my average speed in the first 2 minutes? (2) What is the average rate of change of the function $f(x)$ on the interval $[0, 4]$ and (3) how can you phrase this question to be similarly phrased as the previous two? (4) Set up but do not solve a limit which could be evaluated to find $f'(2)$, and finally (5) interpret the meaning of $f'(2)$ in this example.