Worksheet 6: Chapter 3 (Implicit differentiation and log derivatives)

1. Find the derivative $f'(x)$ two ways: (first way) first use properties of logs to simplify and then take the derivative, and (second way) take the derivative directly using the chain rule. Which way is easier for you?

$$f(x) = \ln \left( \frac{x^2}{1-x} \right)$$

2. Find the equation of the one of the lines tangent to the curve given by the equation

$$3xy - y^2 - x^3 = 1$$

at a point on the curve where $x = -2$. There are two possible answers! Illustrate the problem using software or sketch the situation.
3. Show that the point \((2, 4)\) lies on the curve \(x^3 + y^3 - 9xy = 0\). Then find the tangent and normal to the curve at the point \((2, 4)\). Illustrate the example using software or make a sketch.

4. Use logarithmic differentiation to find the derivative of \(y\) with respect to \(t\) if

\[ y = (\sqrt{t})^t. \]
5. Use either a 30-60-90 or a 45-45-90 reference triangle to evaluate the given expression.

(a) \( \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right) \)

(b) \( \csc^{-1}\left(\frac{-\sqrt{3}}{\sqrt{3}}\right) \)

6. Use the graphs of the trig functions to evaluate the limits. If the limit does not exist write either DNE, +\( \infty \) DNE, or -\( \infty \) DNE whichever is the most appropriate.

(a) \( \lim_{x \to 1} \sin^{-1} x \)

(b) \( \lim_{x \to \infty} \sec^{-1}(x) \)

(c) \( \lim_{x \to 1^-} \cos^{-1}(x) \)
7. Use the inverse trig derivative identities to find \( y' \).

(a) \( y = \cos^{-1}(x^2) \)

(b) \( y = \sec^{-1}(5x) \)