Worksheet #5

1. Let $A$ and $B$ be sets. If $|A| = 5 = |B|$, how many onto functions $f : A \to B$ are there? What if $|A| = 5$ and $|B| = 4$? Can you find a general formula for the case that $|A| = n$, $|B| = m$, where $n > m$? What about $n = m$? And for $n < m$?

2. Six friends walk into a bar . . .
   (a) If each friend hugs each other friend, how much love (many hugs) occur(s)?

   (b) How many different ways are there for the friends to sit at the bar (in a straight line, consecutively)?

   (c) If the friends choose instead to sit at a round table, how many arrangements of seats are there (we consider 2 arrangements the same if each person has the same neighbor to the left)?

   (d) How many arrangements are there if again they choose to sit at a round table, but this time we consider 2 arrangements the same if each person is sitting between the same 2 people?

   (e) Do you see a relationship between (b), (c), and (d)? Explain, and conclude that the numbers in parts (b) and (c) will be even for any number of friends greater or equal to 3.

   (f) Amongst the six friends, there is a couple and an inner-circle of 3 friends (neither member of the couple is a member of the inner-circle). Repeat parts (b),(c), and (d) with the added restriction that the couple sits tandem and the inner-circle sits consecutively.