Math. 2403, Practice Test1

1. (a) Solve the initial value problem

\[ \frac{dx}{dt} = \frac{2}{3x + 1}, \quad x(0) = 3. \]

(b) Draw the associated slope field.
(c) Draw the phase portrait.

2. Solve the initial value problem

\[ \frac{dx}{dt} - \frac{2x}{t} = t^2 \cos t, \quad x(\pi) = 1. \]

3. A falling object is subjected to air resistance that is proportional to the velocity of the object. Suppose that the proportionality constant is equal to \( k \), the object has mass \( m \), and the acceleration due to gravity is equal to \( g \).
(a) Derive an equation governing the velocity \( v \) of the object.
(b) Solve the differential equation and determine the limiting (or terminal) velocity of the object.

4. Find the general solution of the system

\[ \frac{dx}{dt} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x. \]

Draw a phase portrait for the system. The origin is a critical point. Describe its type and stability.

5. Find the critical points of the system

\[ \frac{dx}{dt} = x^2 - 2xy \]
\[ \frac{dy}{dt} = 3xy - y^2. \]