1. Do problems 5 (10 pts), 7 (8 pts) and 10(b) (8 pts) on pages 85-87 of Evans’ book.

2. (8 pts) Use the method of reflections to find Green’s function for the first quadrant \( \Omega = \{(x, y) : x, y > 0\} \).

3. (5 pts) Let \( g \in L^1(\mathbb{R}^n) \), i.e.
\[
\int_{\mathbb{R}^n} |g(y)|dy < +\infty.
\]
Prove that if
\[
u(t, x) = \int_{\mathbb{R}^n} \phi(t, x - y)g(y)dy,
\]
where \( \phi \) is the fundamental solution of the heat equation, then
\[
\lim_{t \to \infty} u(t, x) = 0
\]
uniformly in \( x \in \mathbb{R}^n \). This shows that solutions of the heat equation with initial data in \( L^1(\mathbb{R}^n) \) decay to 0 in the supremum norm.

4. (8 pts) Let \( \Omega \) be an open and bounded subset of \( \mathbb{R}^n \) such that \( \Omega \subset B(0, R) \) for some \( R > 0 \). Let \( T > 0 \). Denote \( Q = \Omega \times (0, T] \), and its parabolic boundary \( \partial_p Q = \overline{Q} \setminus Q \). Let \( f \in C(\overline{Q}) \) and let \( u \in C^{1,2}(\overline{Q}) \) be a solution of
\[
u_t - \Delta u = f \quad \text{in} \ Q.
\]
Show that there exists a constant \( C_R \) (independent of \( T \)) such that
\[
\max_{\overline{Q}} |u| \leq \max_{\partial_p Q} |u| + \frac{C_R}{n} \max_{\overline{Q}} |f|.
\]
(Hint: Look at equations satisfied by \( u \pm (A|x|^2 + B) \) for some properly chosen \( A, B \).)

5. (5 pts) Show that if \( g \) and \( u \) are as in problem #3 then for every \( i = 1, ..., n \)
\[
u_{x_i}(t, x) = \int_{\mathbb{R}^n} \phi_{x_i}(t, x - y)g(y)dy.
\]