Math. 6341, Homework assignment #3. Due on November 18.

Each problem is worth 10 points.

1. Do problems 19(a)(b)(c) and 24 on pages 88-90 of Evans’ book.

2. Let \( u \in C^2([0, +\infty) \times \mathbb{R}^3) \) be a solution of

\[
  u_{tt} - \Delta u = 0 \quad \text{in} \quad \mathbb{R}^3 \times (0, +\infty)
\]

\[
  u(x, 0) = g(x), \quad u_t(x, 0) = h(x) \quad \text{for} \quad x \in \mathbb{R}^3,
\]

where \( f, g \in C^2(\mathbb{R}^3) \) and have compact support. Show that there exists \( C \geq 0 \) such that

\[
  |u(x, t)| \leq \frac{C}{t} \quad \text{for all} \quad (x, t) \in \mathbb{R}^3 \times (0, +\infty).
\]

Hint: Use Kirchhoff’s formula

3. Solve the problem

\[
  u_{tt} - u_{xx} = x^3, \quad \text{in} \quad (0, +\infty) \times (-\infty, +\infty),
\]

\[
  u(0, x) = x^2, \quad u_t(0, x) = x \quad \text{on} \quad (-\infty, +\infty).
\]

4. Use separation of variables to solve the Neumann boundary value problem for Laplace’s equation

\[
  u_{xx} + u_{yy} = 0, \quad \text{for} \quad 0 < x < 1, 0 < y < 1,
\]

\[
  u_y(x, 0) = 0, \quad u_y(x, 1) = x - \frac{1}{2} \quad \text{for} \quad 0 < x < 1,
\]

\[
  u_x(0, y) = 0, \quad u_x(1, y) = 0 \quad \text{for} \quad 0 < y < 1.
\]