1)(a) These graphs are **not** isomorphic.

The first graph has 9 vertices, but the second only has 8.
So, there isn't a bijection between the vertices.

(b) These graphs are isomorphic. Calling the left graph $G_1 = (V, E)$ and the right graph $G_2 = (V_2, E_2)$ we have an isomorphism $\phi : V \rightarrow V_2$ defined by:

\[
\begin{align*}
\phi(1) &= 1 \\
\phi(2) &= 2 \\
\phi(3) &= 5 \\
\phi(4) &= 6 \\
\phi(5) &= 7 \\
\phi(6) &= 4 \\
\phi(7) &= 3
\end{align*}
\]

Visually, we can unwind $G_1$:

\begin{align*}
5 &\rightarrow 4 \\
6 &\rightarrow 3 \\
7 &\rightarrow 2
\end{align*}

$G_1$ & $G_2$ are both $\cong C_7$, a 7-cycle.

(c) No isomorphism exists.

In the first graph, the degree 4 vertices aren't adjacent, but in the second they are.

2.) Exercise 5.9 #7

**Claim:** $A \cong C$, and no other pair is isomorphic.

**Proof:** $D$ has a vertex of degree 1, unlike any other graph. Thus, $D$ isn't isomorphic to the others.

- $A \cong C$: There is an isomorphism $\phi$ defined by:

\[
\begin{align*}
\phi(v_1) &= w_1 \\
\phi(v_2) &= w_2 \\
\phi(v_3) &= w_3 \\
\phi(v_4) &= w_4 \\
\phi(v_5) &= w_5
\end{align*}
\]

Corresponding to $A$:

\[
\begin{align*}
v_1 &\rightarrow v_2 \\
v_3 &\rightarrow v_4 \\
v_5 &\rightarrow v_6
\end{align*}
\]
Next, because isomorphisms preserve neighbors, they must also preserve paths & cycles. 

(if \( x_1 \equiv x_2 \equiv x_3 \), then \( \phi(x_1) \equiv \phi(x_2) \equiv \phi(x_3) \) )

B has no cycle of length 3, but A & C do. Thus, B \# A, B \# C.

\[(y_2, y_3, y_6) \quad (\overline{w}_3, \overline{w}_4, \overline{w}_5)\]

This proves our claim.

3) Exercise 5.9 #8

Let's use the old Eulerian circuit algorithm.

\[C=(4)\]

Loop 1: \[C=(1, 4, 8, 3, 2, 7, 12, 2, 9, 3, 5, 6, 7)\]

(marked in red.)

Loop 2: The first vx with unused edges in \(C\) is 4.

New path:

\[(4, 9, 10, 2, 11, 4)\] (marked in green)

Store this into \(C\) where \(H\) was before:

\[(1, 4, 9, 10, 2, 11, 4, 8, 3, 2, 7, 12, 2, 9, 3, 5, 6, 7)\] (Ans)

(All edges used now)

4) Exercise 5.9 #10

Vertices 12 & 5 have odd degree, but all other vx's have even degree. Adding one edge between 12 & 5 makes all vx's have even degree, resulting in the graph becoming Eulerian.

5) This statement is true.

Proof:

\[\rightarrow\] If an Eulerian trail exists, we must enter & leave any particular vx the same number of times -- except for perhaps the first and last vx's -- meaning that all but at most 2 vx's have even degree, so at most 2 have odd degree.

\[\rightarrow\] On the flip side, consider a graph \(G\) with at most 2 vx's of odd degree. Because the #vx's of odd degree in any graph is even, \(G\) has either 2 or 0 vx's of odd degree.
If $G$ has 0 odd degree vxs, it has an Euler circuit, which is an Eulerian trail.

If $G$ has 2 odd degree vxs, form a graph $G'$ by adding an edge between these vxs. $G'$ has no odd degree vxs, and so it has an Eulerian circuit. Cut out the edge we just added, and we're left with an Eulerian trail in $G$.

(Math notation wise, if $(x_1, x_2, ..., x_k, x_{k+1}, ..., x_n)$ is a circuit in $G'$, and $x_k$ and $x_{k+1}$ in $G$, then $(x_{k+1}, ..., x_n, x_1, x_2, ..., x_k)$ is an Eulerian trail in $G$.)

6) Exercise 5.9 #12

How many edges are possible in a graph with 17 vxs?

$$\binom{17}{2} = \frac{17 	imes 16}{2} = 136$$

Thus, the graph in question has all but 7 possible edges.

Even if all 7 missing edges were touching 1 vx, it would still have

$$17 - 1 - 7 = 9$$

neighbors. Thus, every vx has at least 9 neighbors, and

$$9 = \left\lceil \frac{17}{2} \right\rceil.$$ Thus, by Theorem 5.5 (Dirac), the graph must be Hamiltonian.

7) Exercise 5.9 #14

The vxs in red form a 4-clique, showing we need at least 4 colors. I found a 4-coloring (as shown), so the graph's chromatic number is 4.
8.) a) 2 (no triangles)  b) 3 (a triangle, no)  c) 4 - all vs connected,

9) There's only one way to color the top up to switching 1, 2, & 3.

Needs a 4th color.

So, \( \chi(G) = 4 \), \( w(G) = 3 \) because it contains \( \Delta \)'s but no 4 vs are all attached.

10.) Exercise 5.9#28

It is planar, with a few small adjustments.

j & i were pushed down, and edge gb was relocated.