1. Prove that for all $n \geq 1$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Do not turn these in. They serve as hints.

Show that for all positive integers $n$, $7^n - 4^n$ is divisible by 3.

Show that for all positive integers $n$, $9^n - 5^n$ is divisible by 4.

2. It turns out that if $a$ and $b$ are positive integers with $a > b + 1$, then there is a positive integer $M > 1$ such that $a^n - b^n$ is divisible by $M$ for all positive integers $n$. Determine $M$ in terms of $a$ and $b$ and prove that it is a divisor of $a^n - b^n$ for all positive integers $n$.

3. Use mathematical induction to prove that for all integers $n \geq 1$,

$$n^3 + (n + 1)^3 + (n + 2)^3$$

is divisible by 9.

4. Consider the recursion given by $f(n) = f(n - 1) + f(n - 2)$ for $n \geq 3$ with $f(1) = f(2) = 1$. Show that $f(n)$ is divisible by 3 if and only if $n$ is divisible by 4.

5. Suppose that $x \in \mathbb{R}$ and $x > -1$. Prove that for all integers $n \geq 0$, $(1 + x)^n \geq 1 + nx$. 