Variation of Parameters. Reduction of Order.

Consider

\[ x^2 y'' + 2xy' - 2y = x. \]

1. Verify that \( y_1(x) = x \) is a particular solution of the corresponding homogeneous equation

\[ x^2 y'' + 2xy' - 2y = 0. \]

2. Find all solutions of the homogeneous equation.

3. Find all solutions of the nonhomogeneous equation.

ANSWERS:

1. DIY (just plug the function in the homogeneous equation to see whether the equation is satisfied or not)

2. \( C_1x + C_2x^{-2} \) where \( C_1, C_2 \) are arbitrary constants

3. \( \frac{1}{3} x \ln |x| - \frac{1}{9} x + C_1x + C_2x^{-2} \) where \( C_1, C_2 \) are arbitrary constants