1. (10 points) In this problem, you are given the following four differential equations:

\( \frac{dy}{dx} = x(y - 1); \quad (2) \frac{dy}{dx} = 1 - y^2; \quad (3) \frac{dy}{dx} = (x - 2)^2 + y^2; \quad (4) \frac{dy}{dx} = \frac{2 - x}{y}. \)

(a) (2 points x 3) Which of the above equations is (are) linear? 

Which of the above equations is (are) autonomous? 

Which of the above equations is (are) separable? 

(b) (2 points x 2) For each of the following two direction fields, which equation does it correspond to? Choose the correct one from the four equations (1)–(4) above.

2. (10 points) Solve the initial value problem \( y' = 2y^2 + x^2y^2, \ y(0) = 1. \) Your final solution should be in explicit form.

**Solution:** The equation is separable since it can be written as \( \frac{dy}{dx} = (2 + x^2)y^2. \) We move all \( x \) to the right hand side, all \( y \) to the left hand side, then integrate both sides:

\[
\int \frac{1}{y^2} dy = \int (2 + x^2) dx + C
\]

\[-\frac{1}{y} = 2x + \frac{x^3}{3} + C.
\]

Since \( y(0) = 1 \), plugging in \( x = 0, \ y = 1 \) into the above equation gives us \( C = -1 \). Hence the solution to the IVP is

\[-\frac{1}{y} = 2x + \frac{x^3}{3} - 1,
\]

which becomes

\[ y = -\frac{1}{2x + \frac{x^3}{3} - 1} \]

in explicit form.