1. (6 points) Assume that \((x_0, y_0)\) is a critical point of the system \[
\begin{align*}
x'(t) &= F(x, y) \\
y'(t) &= G(x, y),
\end{align*}
\]
and the approximating linear system near \((x_0, y_0)\) is \(u' = Au\). In each of the following cases, pick the correct answer from (A)-(G) below.

- If the eigenvalues of \(A\) are \(\lambda_1 = 3, \lambda_2 = -4\), then we have (C).
- If the eigenvalues of \(A\) are \(\lambda_1 = 5i, \lambda_2 = -5i\), then we have (G).
- If the eigenvalues of \(A\) are \(\lambda_1 = 2 - 3i, \lambda_2 = 2 + 3i\), then we have (D).

(A) \((x_0, y_0)\) must be a nodal source for the nonlinear system.
(B) \((x_0, y_0)\) must be a nodal sink for the nonlinear system.
(C) \((x_0, y_0)\) must be a saddle point for the nonlinear system.
(D) \((x_0, y_0)\) must be a spiral source for the nonlinear system.
(E) \((x_0, y_0)\) must be a spiral sink for the nonlinear system.
(F) \((x_0, y_0)\) must be a center for the nonlinear system.
(G) Either the type or stability of \((x_0, y_0)\) is inconclusive for the nonlinear system.

2. (14 points) For the system
\[
\begin{align*}
x'(t) &= y - 1 \\
y'(t) &= x^2 - y^2,
\end{align*}
\]
Find all of its critical points, and determine the type and stability for each of them.

**Solution:** Setting both right hand sides equal to zero, we know that there are two critical points: \((1, 1)\) and \((-1, 1)\). The Jacobian at \((x, y)\) is 
\[
J(x, y) = \begin{pmatrix} 0 & 1 \\ 2x & -2y \end{pmatrix}.
\]
Plugging in the two critical points into the Jacobian respectively, we have 
\[
J(1, 1) = \begin{pmatrix} 0 & 1 \\ 2 & -2 \end{pmatrix}, \quad \text{and} \quad J(-1, 1) = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix}.
\]
So near \((1, 1)\), the approximating linear system is \(u' = \begin{pmatrix} 0 & 1 \\ 2 & -2 \end{pmatrix} u\), and its eigenvalues are \(\lambda = -1 \pm \sqrt{3}\). Therefore \((1, 1)\) is an (unstable) saddle point.

Near \((-1, 1)\), the approximating linear system is \(u' = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} u\), and its eigenvalues are \(\lambda = -1 \pm i\). Therefore \((-1, 1)\) is a (stable) spiral sink.