Below are all the formulas you will need for this quiz: \[ \mathcal{L}\{1\} = \frac{1}{s}, \quad \mathcal{L}\{e^{at}\} = \frac{1}{s - a}, \]
\[ \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \text{ (for positive integer } n), \quad \mathcal{L}\{(f^{(n)}(t))\} = s^n F(s) - s^{n-1} f(0) - \ldots - f^{(n-1)}(0). \]

1. (11 points) Find the Laplace transform of the solution of the following initial value problem. You only need to find \( \mathcal{L}\{y(t)\} \), and there is no need to take the inverse Laplace transform.

\[ y'' + 2y = t^2 + 4e^{3t}, \quad y(0) = 0, y'(0) = 1. \]

**Solution:** Taking the Laplace transform on both sides and applying the above formulas, we have

\[ s^2 Y(s) - sy(0) - y'(0) + 2Y(s) = \frac{2}{s^3} + \frac{4}{s - 3}. \]

Plugging in \( y(0) = 0, y'(0) = 5 \) gives

\[ s^2 Y(s) - 1 + 2Y(s) = \frac{2}{s^3} + \frac{4}{s - 3}, \]

hence we have

\[ \mathcal{L}\{y(t)\} = Y(s) = \frac{\frac{2}{s^3} + \frac{4}{s - 3} + 1}{s^2 + 2} = \frac{s^4 + s^3 + 2s - 6}{s^2(s - 3)(s^2 + 2)}. \]

2. (4 points)

- Let \( f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ \frac{1}{(t-1)^2}, & 1 \leq t < 2 \\ 0, & 2 \leq t \leq 3 \end{cases} \). Is \( f \) piecewise continuous in \([0, 3]\)?
  
  **Yes** \quad **No**

  (By definition in page 299, \( f \) is piecewise continuous only when \( f \) approaches a finite limit as \( t \) approaches both endpoints of each subinterval)

- Let \( f(t) = 2e^{3t} \sin(5t) \). Is \( f \) of exponential order?
  
  **Yes** \quad **No**

  (We have \( |f| \leq 2e^{5t} \), so it is of exponential order by definition in page 300.)

3. (5 points) Let

\[ F(s) = \frac{-s^2 + 3s - 3}{s^2(s + 3)(s^2 + 4)}. \]

Write down the correct form of its partial fraction expansion. Do NOT solve for the coefficients.

\[ F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s + 3} + \frac{Ds + E}{s^2 + 4}. \]