Flat fully augmented links are determined by their complements

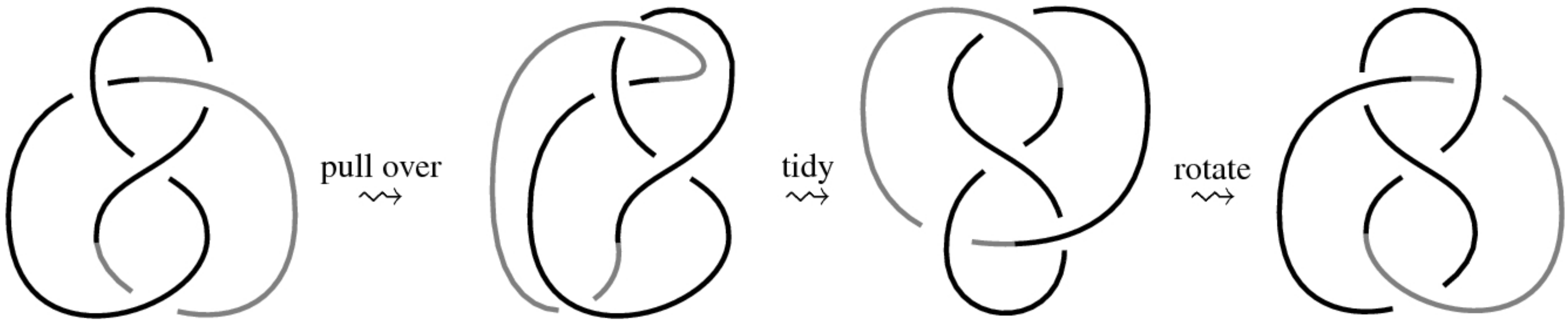
Christian Millichap joint with Rollie Trapp (CSUSB)

Furman University

Tech Topology Conference

Equivalence of links

Two links, L and L', are equivalent if there is an ambient isotopy between them in $\mathbb{S}^3$.



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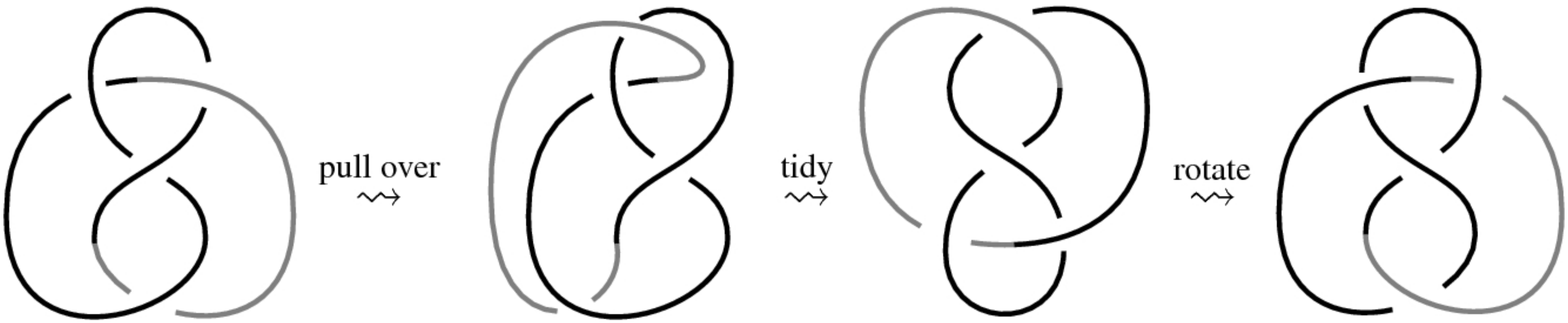

Figure: The figure–8 knot is equivalent to its mirror image.

An equivalence of links L and L' induces a homeomorphism between their respective complements, $M_L = \mathbb{S}^3 \setminus L$ and $M_{L'} = \mathbb{S}^3 \setminus L'$.

Links and their complements

Question: If two link complements are homeomorphic, then are the corresponding links equivalent?

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Answer: Not always!

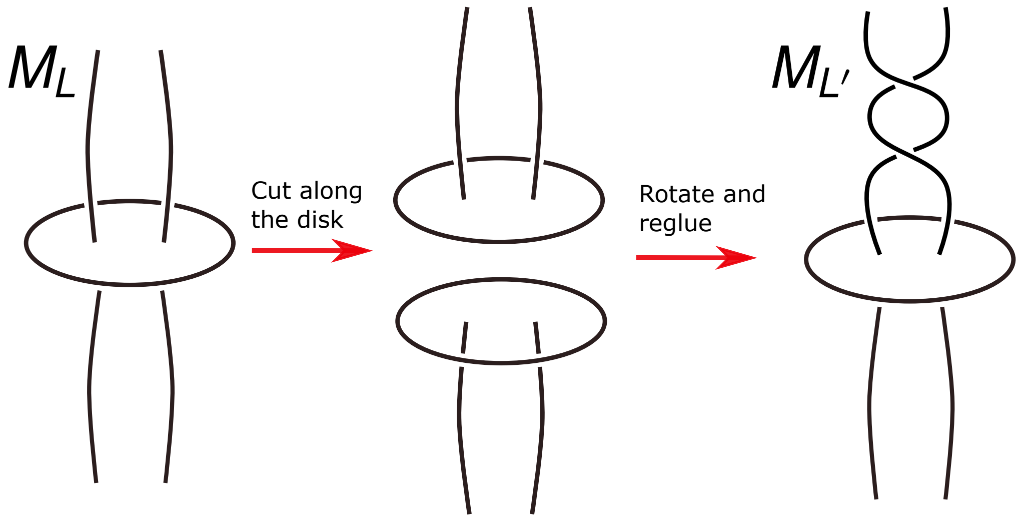

Figure: Two links whose complements are homeomorphic via a Dehn-twist along a disk bound by an unknotted component. This procedure frequently results in links that are not equivalent. The local picture of the corresponding links is given here.

Links and Their Complements

Question: Is there a set of links $\mathcal{S}$ such that if $L_1, L_2 \in \mathcal{S}$ and $\mathbb{S}^3 \setminus L_1$ is homeomorphic to $\mathbb{S}^3 \setminus L_2$, then L_1 is equivalent to L_2? (Links in the set $\mathcal{S}$ are determined by their complements.)

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Any others? YES! Flat fully augmented links (flat FALs).

Fully Augmented Links (FALS)

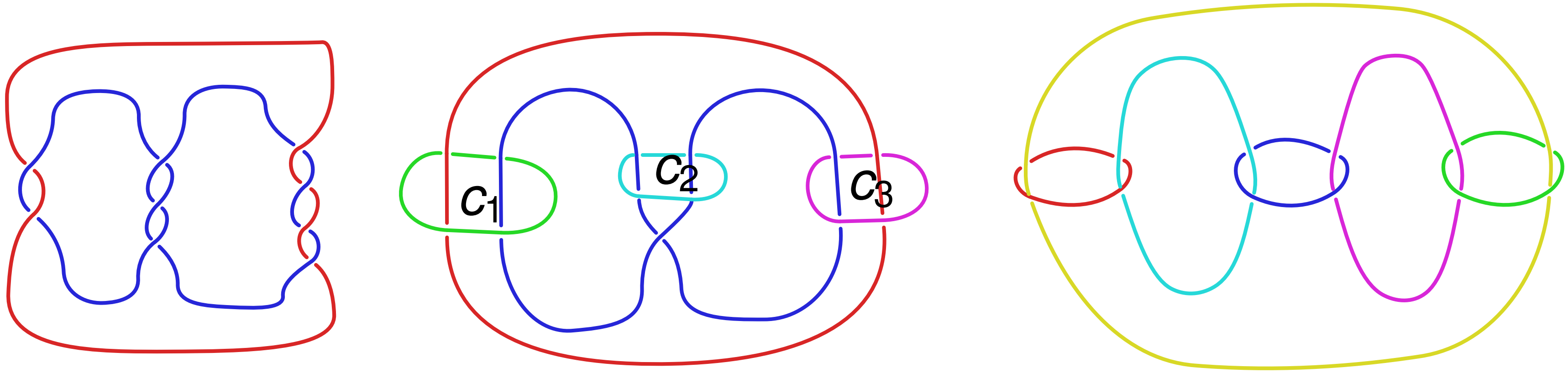
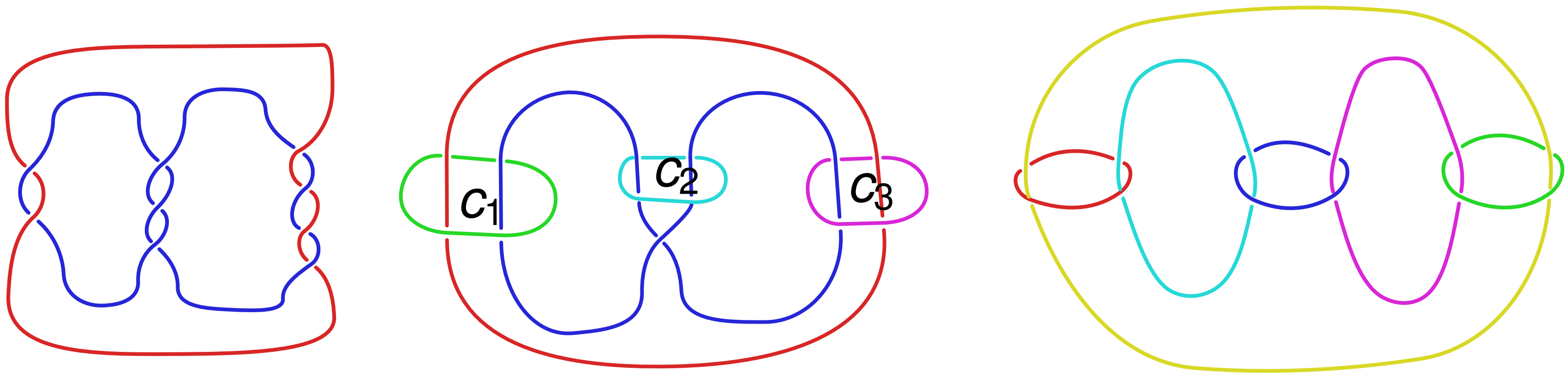


Figure: A link diagram on the left, its corresponding FAL diagram in the middle, and its corresponding flat FAL on the right. **Crossing circles** labeled by c_i in the middle diagram.

Fully Augmented Links (FALS)



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Any reasonable FAL complement decomposes into a pair of identical right-angled ideal hyperbolic polyhedra with totally geodesic faces.

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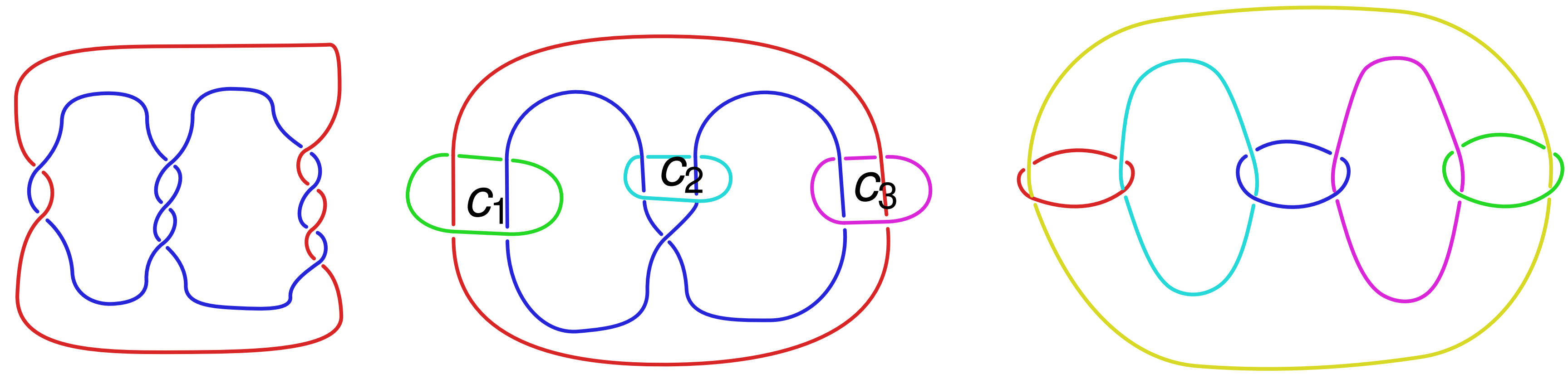

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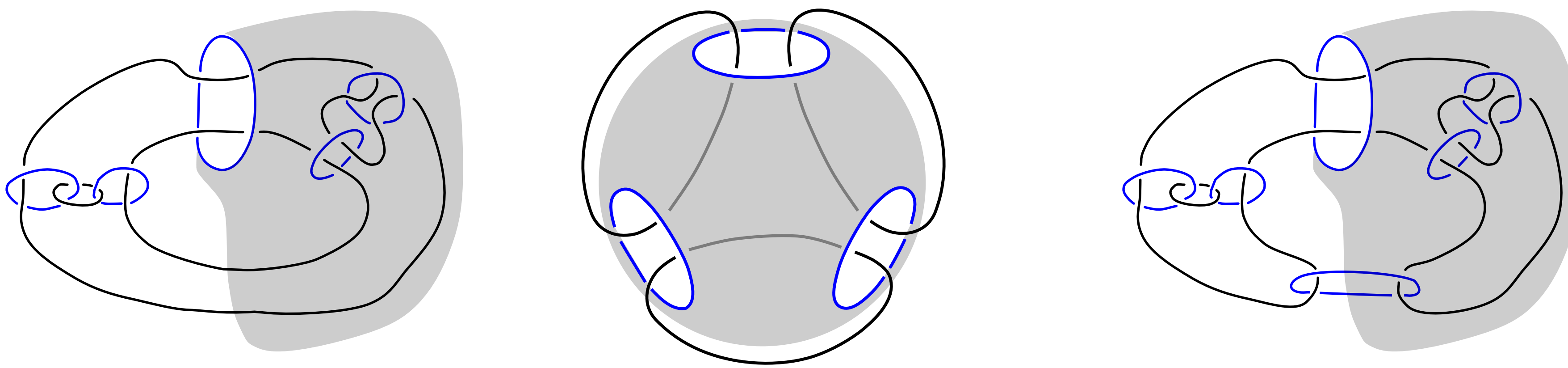
Brief Background and History:

- Geometry of augmented links first studied by Adams (1984).
- Geometric decomposition into ideal polyhedra (Agol & D. Thurston, 2004).
- Geometry and topology of FALs (Purcell, Futer–Purcell, Flint, Trapp and REU students, Hoffman–Worden, others).

Flat FALs are determined by their complements

Technique: Leverage the topology and geometry of totally geodesic surfaces and cusps in a flat FAL complement.



(*a*) A crossing disk (*b*) A longitudinal disk (*c*) A singly-separated disk

Figure: Types of non-reflection, thrice-punctured spheres

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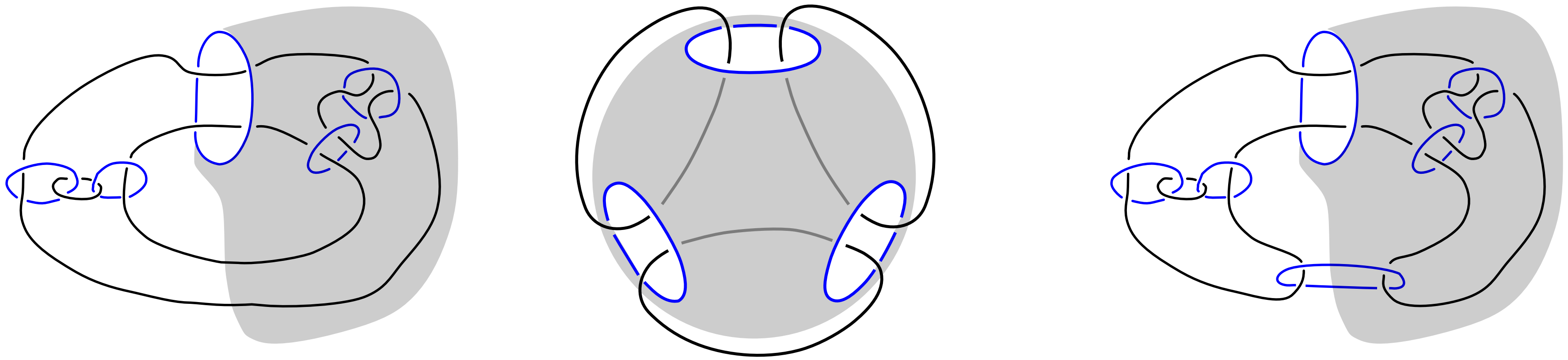

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Figure: Types of non-reflection, thrice-punctured spheres

Theorem (Millichap–Trapp)

Let F and F' be two flat FALs. Then $\mathbb{S}^3 \setminus F$ and $\mathbb{S}^3 \setminus F'$ are homeomorphic if and only if F and F' are equivalent as links.

Thank You!

My contact info: Christian.Millichap@furman.edu

Preprint will be up on arXiv soon!

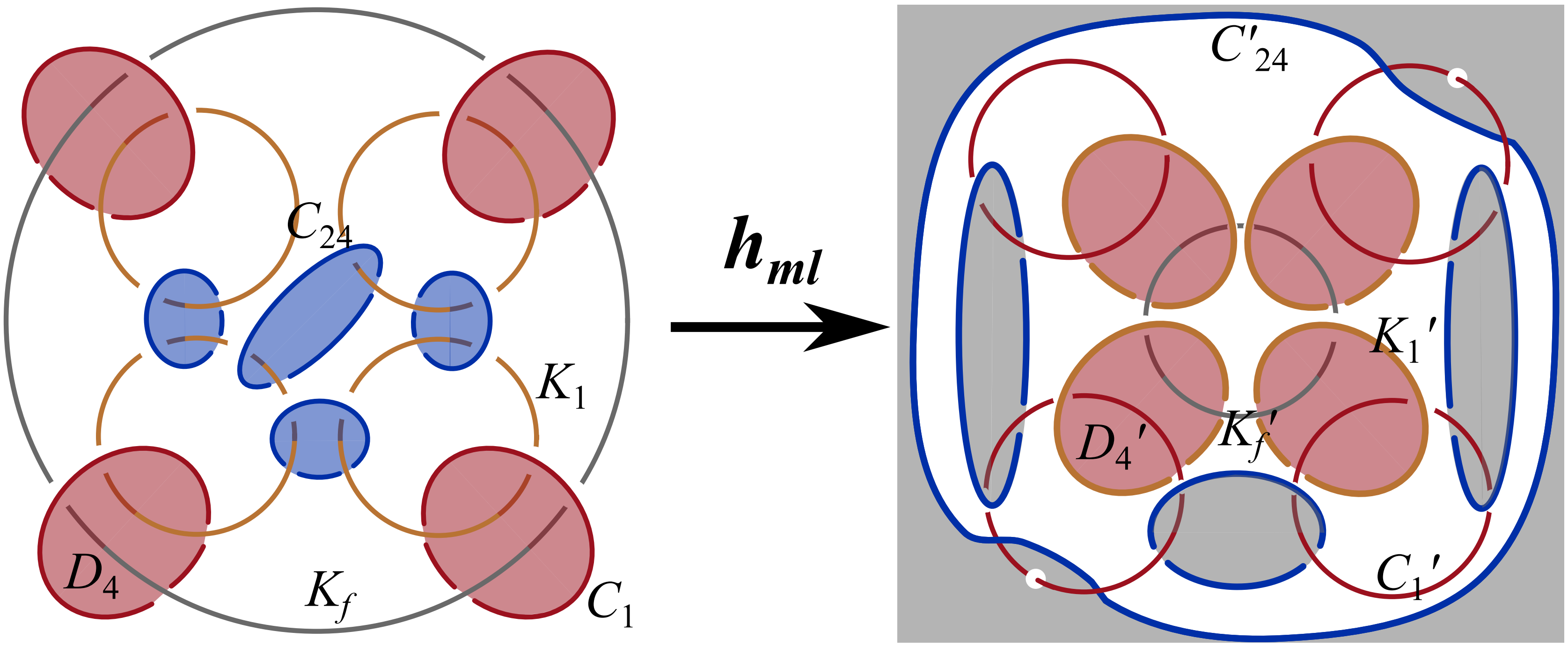
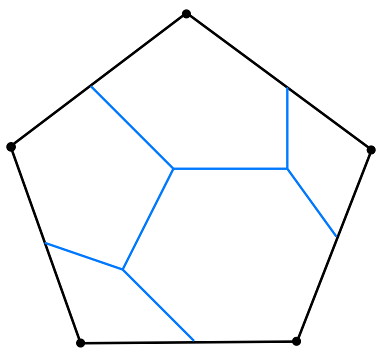


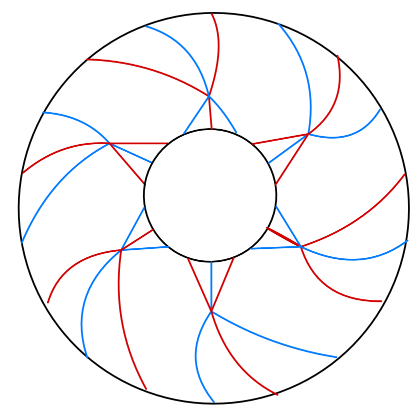
Figure: A homeomorphism between FAL complements. The resulting links are equivalent!

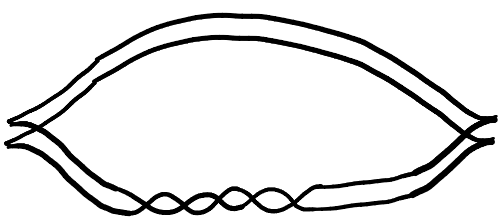
Legendrian Loops and Mapping Class Groups

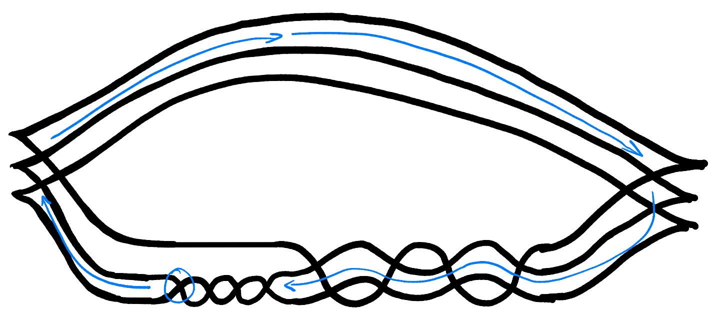


James Hughes (UC Davis)

@ Tech Topology Conference

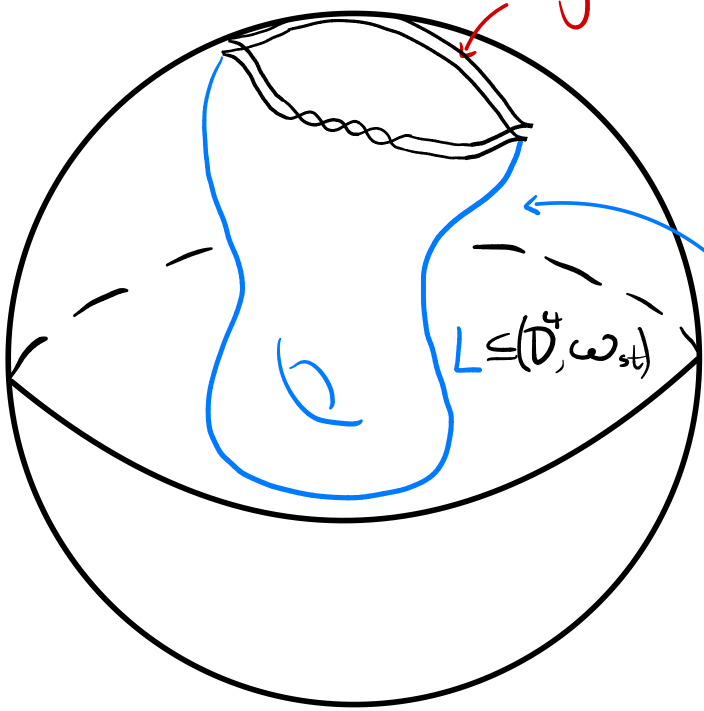






Legendrian Links + Lagrangian Fillings

Legendrian link $\Lambda \subseteq \partial(D^4, \omega_{st}) \cong (S^3, \xi_{st})$



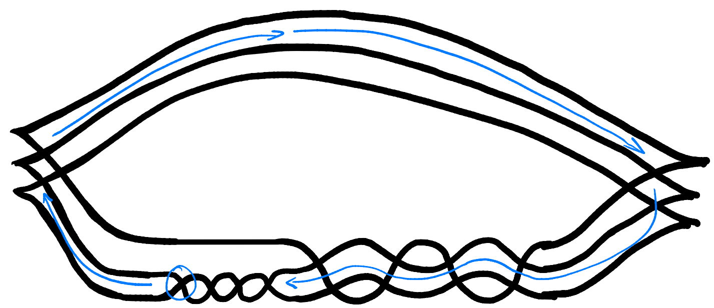
$$T_x\Lambda \subseteq \xi_{st} := \ker(\underbrace{dz - y\,dx}_{\alpha_{st}})$$

Exact Lagrangian filling L:

- $\partial L = \Lambda$
- $\omega_{st}|_L \equiv 0$
- $\alpha_{st}|_L$ is exact

Legendrian Loops

Legendrian loops act on the set of exact Lagrangian fillings by concatenation.



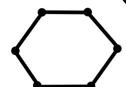
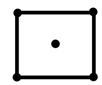
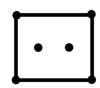
Thm: (Casals-Gao '20) The Legendrian torus links $\Lambda(n,m)$ $n \geq 3$, $m \geq 6$ admit infinitely many Lagrangian fillings.

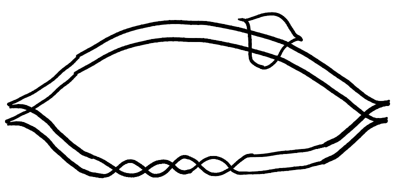
Invariants

Contact geometry		(Cluster) algebraic invariants
• Legendrian link Λ (braid positive)	$\longrightarrow$	Algebraic variety $X(\Lambda)$
• Exact Lagrangian filling L of Λ	$\longrightarrow$	Toric chart $(\mathbb{C}^\times)^{b_1(L)} \subseteq X(\Lambda)$
• Legendrian loop	$\longrightarrow$	(cluster) automorphism of $X(\Lambda)$

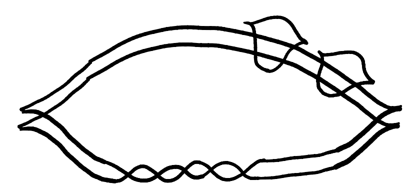
Legendrian Loops (Revisited)

Thm (H. '22) The following cluster automorphism groups are generated by Legendrian loops and a single contactomorphism:

- $\mathrm{Aut}(X(\Lambda(2,n))) \cong \mathrm{MCG}($  $) \cong \mathbb{Z}_{n+2}$
- $\mathrm{Aut}(X(\Lambda(D_n))) \cong \mathrm{MCG}^{\bowtie}($  $) \cong \mathbb{Z}_n \times \mathbb{Z}_2$
- $\mathrm{Aut}(X(\Lambda(\tilde{D}_n))) \cong \mathrm{MCG}^{\bowtie}($  $) \cong \langle \sigma_1, \sigma_2, \tau \mid \sim \rangle$
- $\mathrm{Aut}^{*}(X(\Lambda(k,n))) \cong \langle \sigma_1, \ldots, \sigma_n, \rho, \tau \mid \sim \rangle$



$\Lambda(D_n)$



$\Lambda(\tilde{D}_n)$

* conjectural for some values of k, n

Plat Representations of the Unknot

Deepisha Solanki

University at Buffalo

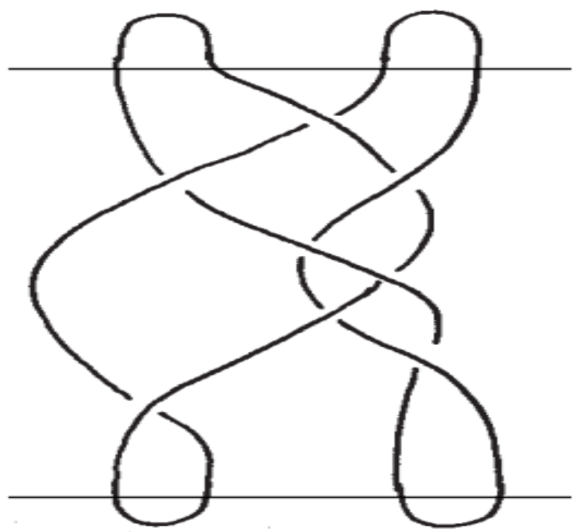
9th December 2022, Tech Topology Conference

Making knots from braids: Plats

We have a method of constructing knots from braids, as outlined below:

Definition

A $2n$-braid completed by $2n$ simple arcs, to make a link, as shown in the figure below, is called a **plat** or **2n-plat**.



Definition

n is then called the **bridge index** of the **plat**.

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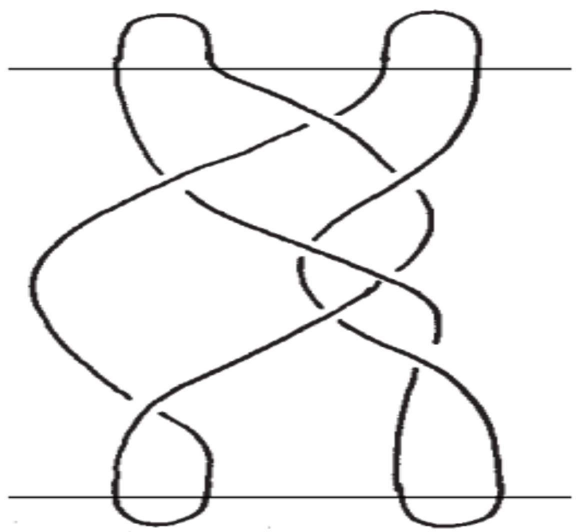

Definition

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Birman's Result

Theorem

Birman, 1976

Any two plat representatives of a knot K are related to each other via the following moves, which take plats to plats

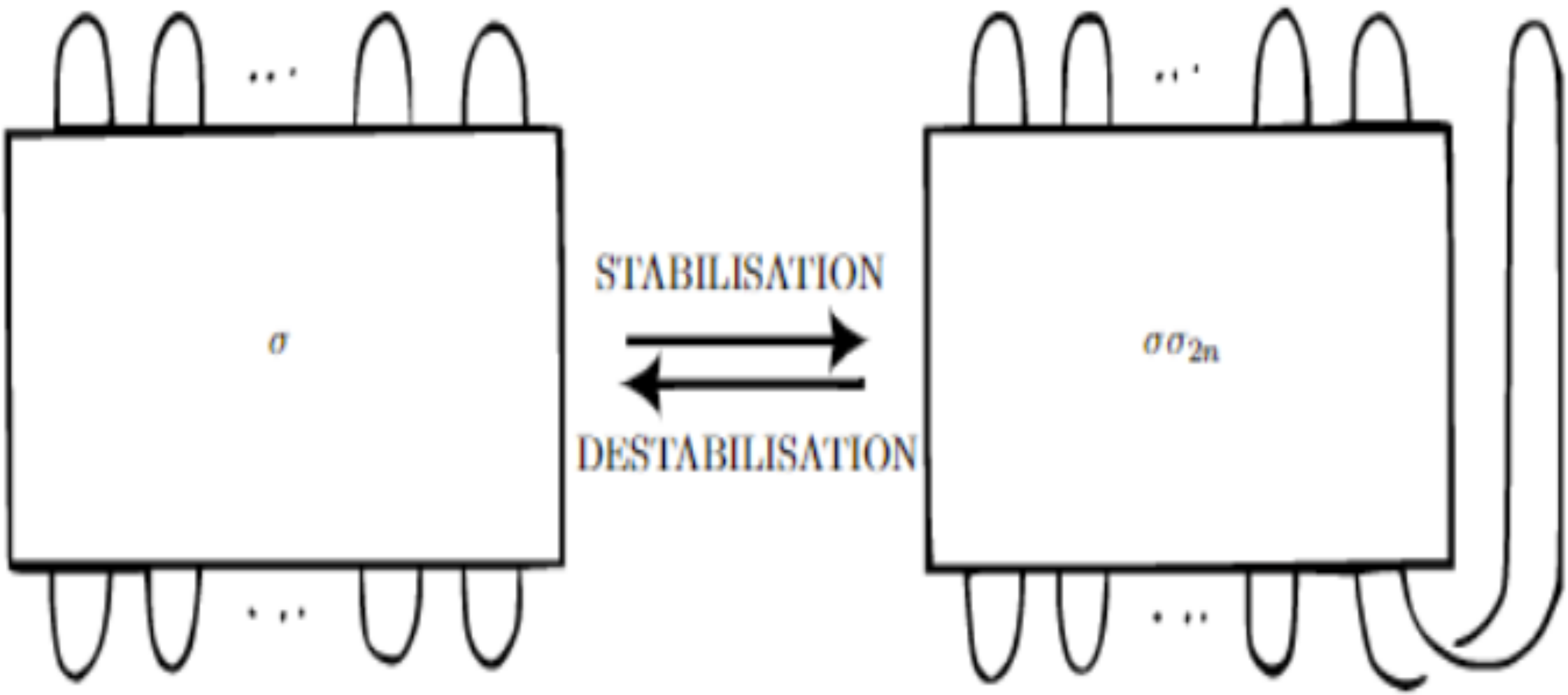
(i) Braid isotopies

(ii) Double coset moves

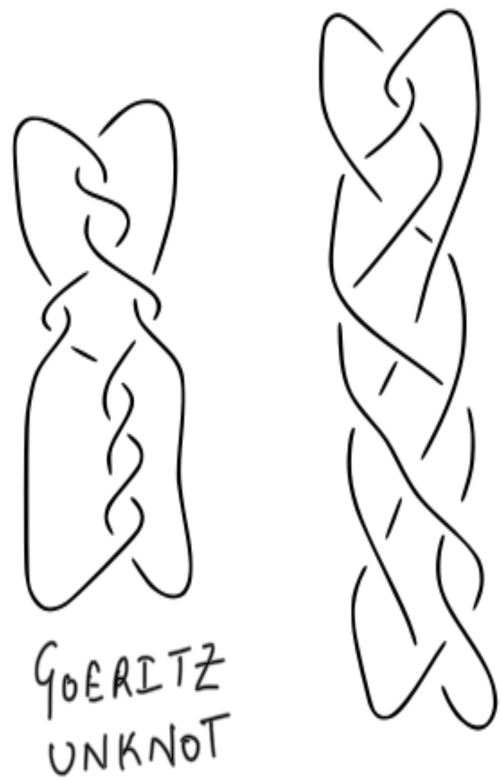
(iii) Addition or deletion of a trivial loop (stabilisation or destabilisation)

Stabilisation

Stabilising a $2n$ plat means adding a trivial loop to the plat, thus increasing its bridge index by 1, as shown below:



Why stabilisation is bad!



- These are plat representations of the **unknot** which can not be simplified to the standard 0-crossing representation of the unknot without stabilisation!!
- Connect summing these plats to a plat diagram of any knot, we can observe the same phenomenon for that knot class, thus making the need to stabilise all pervasive.

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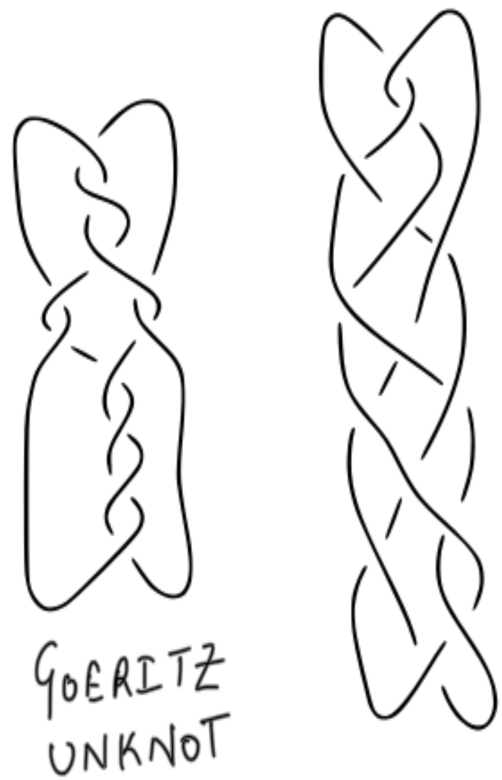

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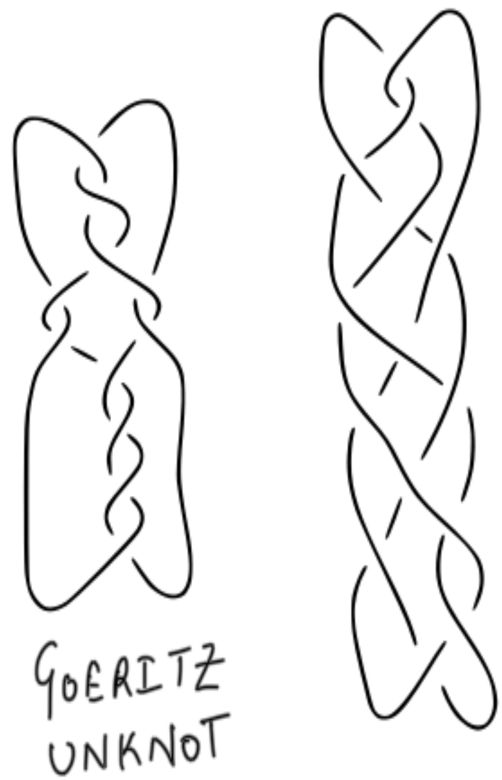

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The main result: Avoiding Stabilisation

Theorem

Any *plat representative of the* ***unlink*** *can be simplified to the standard, 0-crossing diagram of the unlink via the following* ***non index-increasing*** *moves:*

(i) Braid isotopies

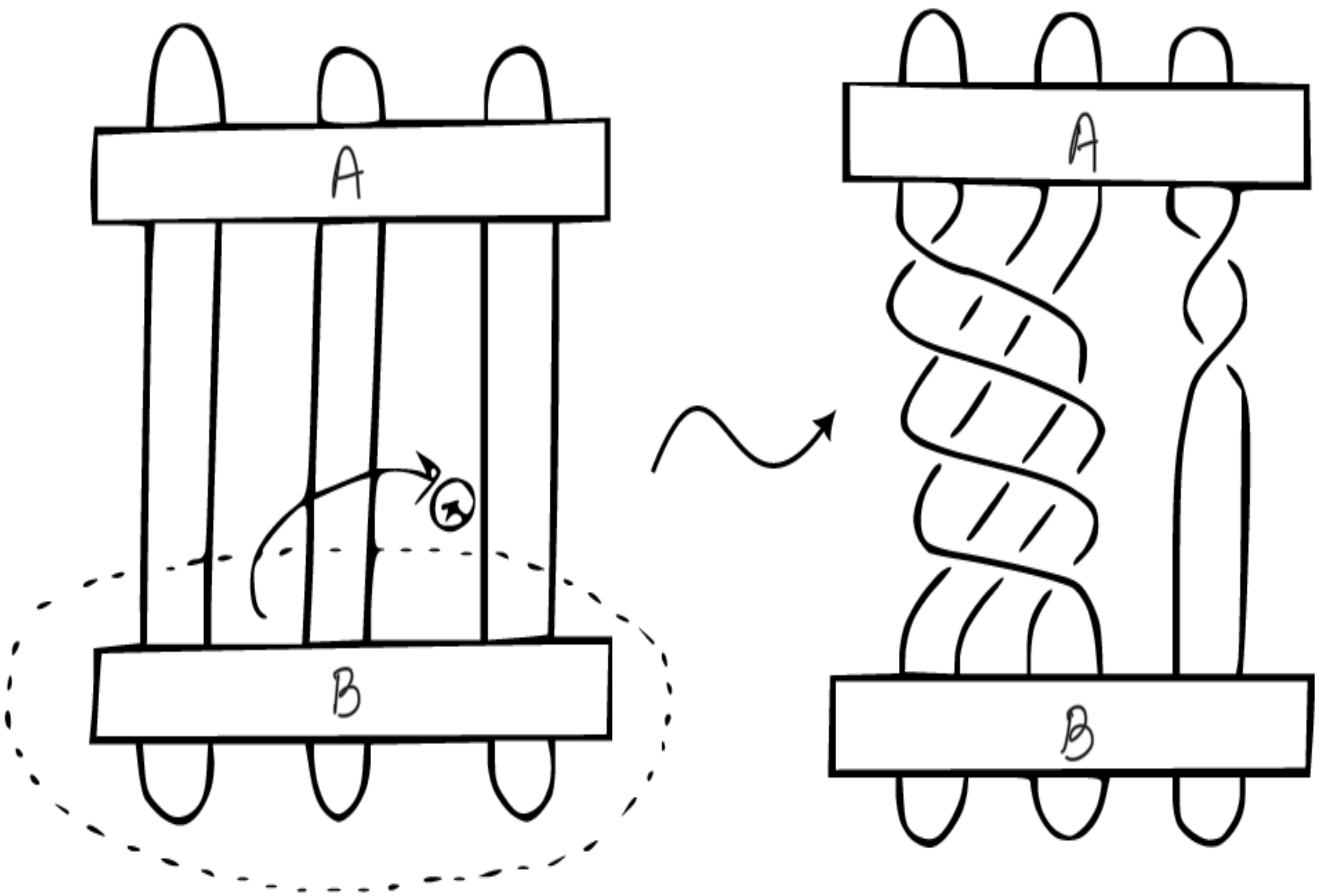
(ii) Double coset moves

(iii) Destabilisation (deletion of a loop)

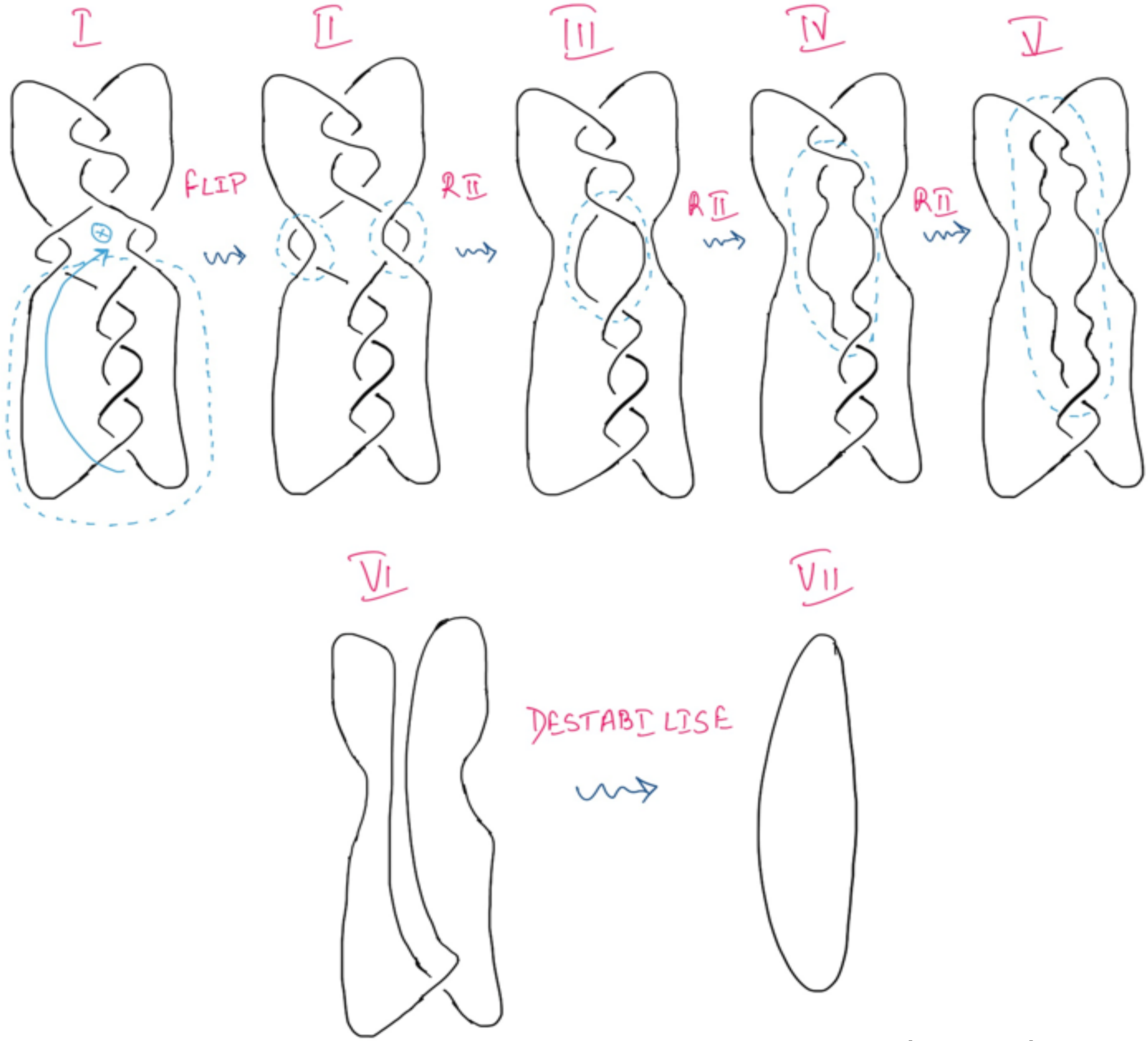
(iv) ***The flip move***

The flip move

We found a new move, called the **flip move**, which obviates the need to stabilise.



The flip move in action



Thank you!

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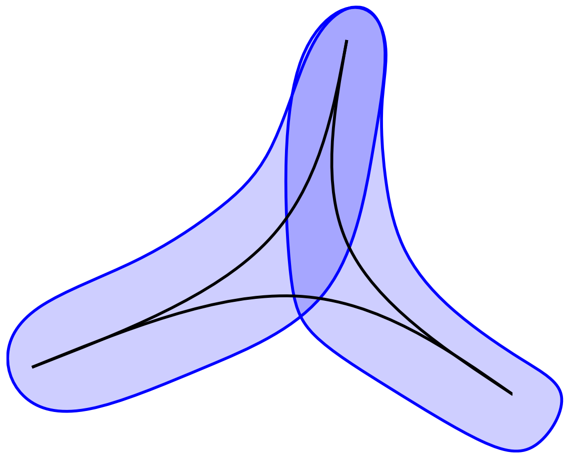
deepisha@buffalo.edu

Purely pseudo-Anosov subgroups of fibered 3-manifold groups

w/ Chris Leininger

Hyperbolic group:

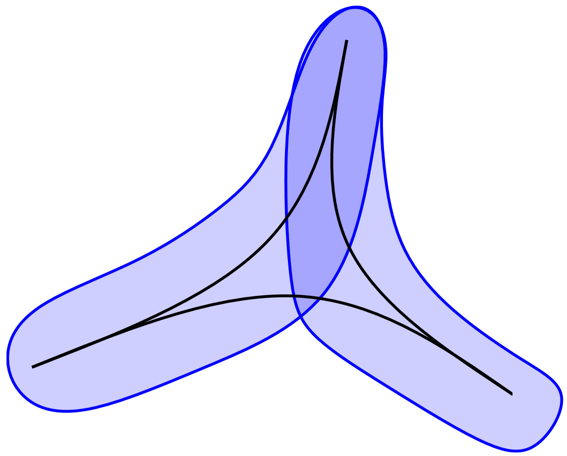
Cayley graph with thin triangles



Lemma (Gromov)

G hyperbolic $\Longrightarrow$ no BS subgroups

$\mathrm{BS}(n,m) = \langle a, b \mid ba^n b^{-1} = a^m \rangle$



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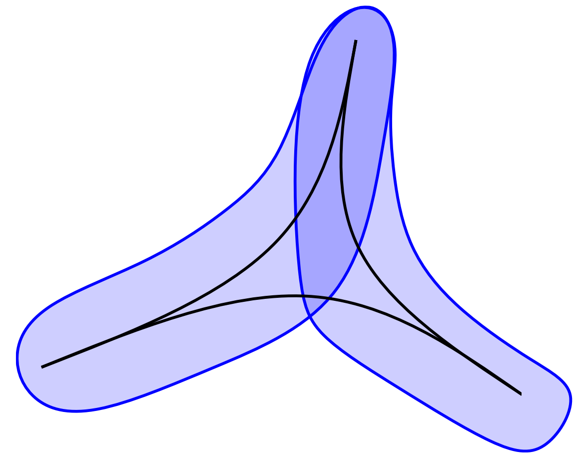

Gromov's "no BS" Conjecture:

Does no BS subgroups $\Longrightarrow$ hyperbolic?

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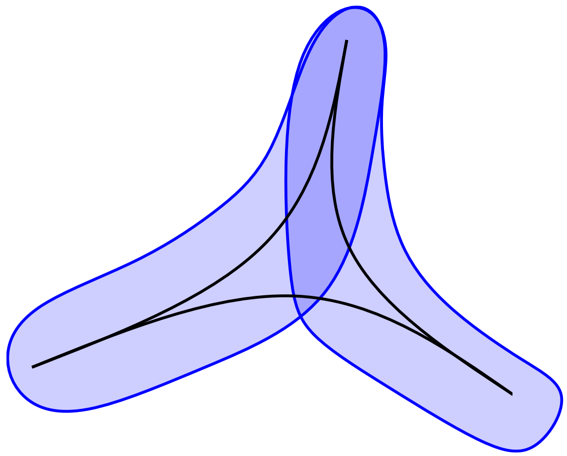

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Finitely Presented

Brady

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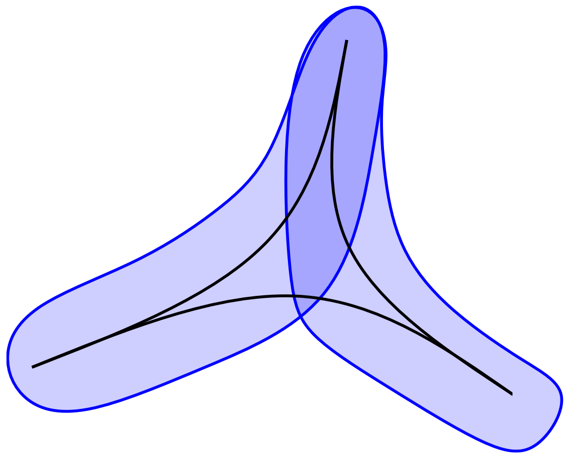

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False	True
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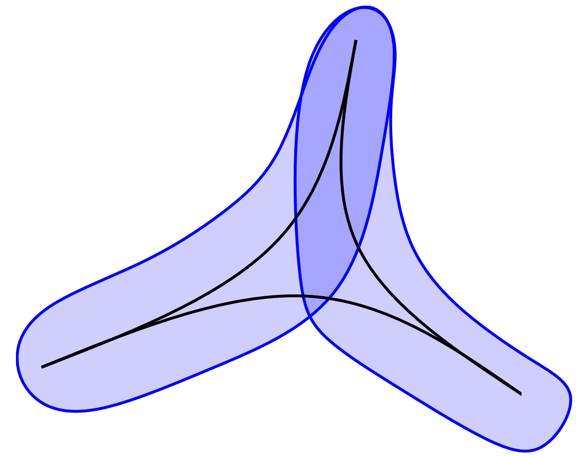

Gromov's "no BS" Conjecture:

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False	True	Open
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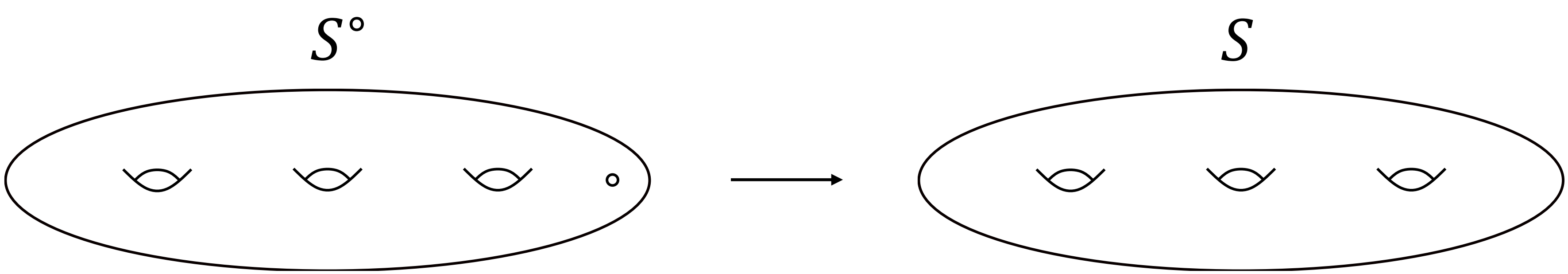
Brady	Perelman	

Birman Exact Sequence

$$1 \longrightarrow \pi_1(S) \longrightarrow \mathrm{MCG}(S^\circ) \longrightarrow \mathrm{MCG}(S) \longrightarrow 1$$



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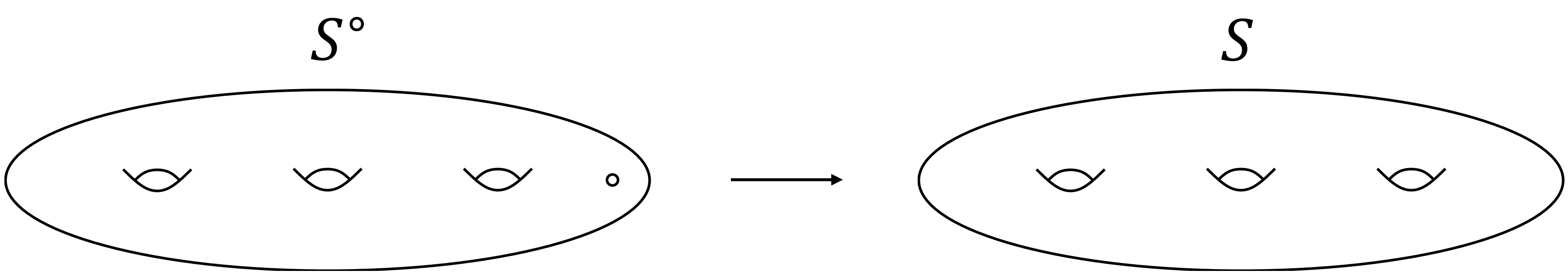
$$\vee$$

$$G$$



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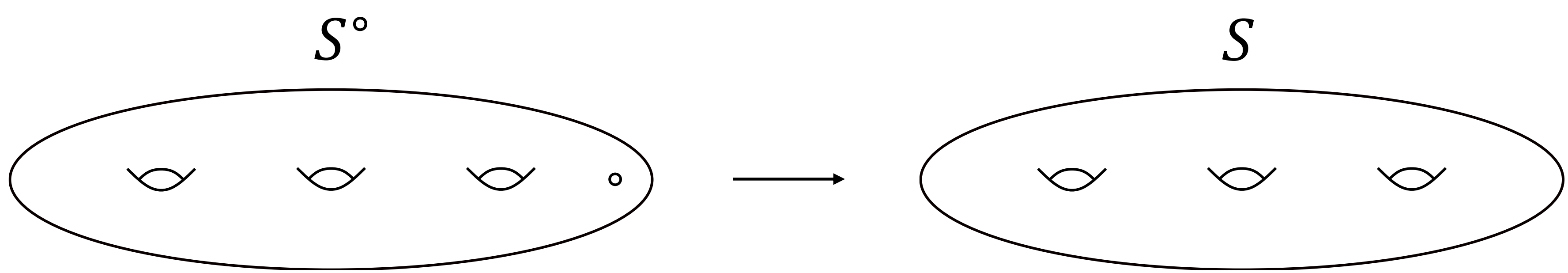

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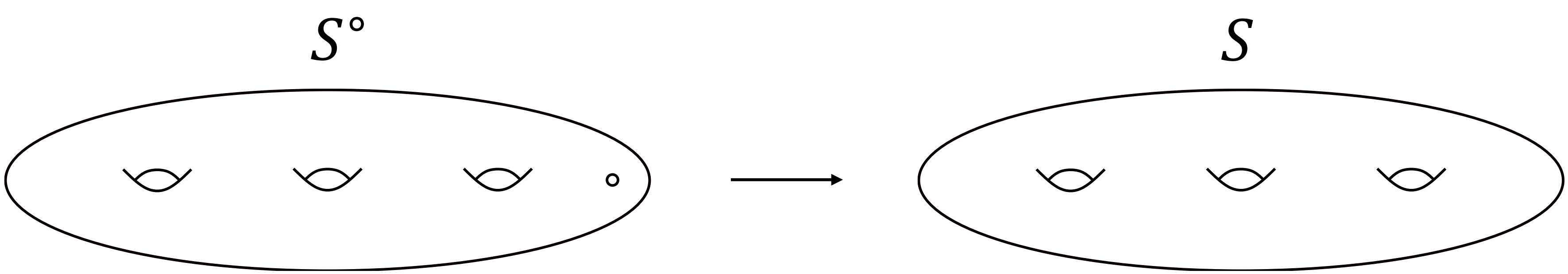

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mixing dynamics

E no BS subgroups $\Leftrightarrow$ G purely pseudo-Anosov

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nice geometry

E hyperbolic $\Leftrightarrow$ G convex cocompact

Farb–Mosher & Hamenstädt

nice geometry

mixing dynamics

Convex Cocompact $\Longrightarrow$ finitely generated + purely pA

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Farb–Mosher: Does fin. gen. + purely pA $\Longrightarrow$ convex cocompact?

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Equivalent to
"no BS Conjure" for E

E no BS subgroups $\Longleftrightarrow$ G purely pseudo-Anosov

E hyperbolic $\Longleftrightarrow$ G convex cocompact

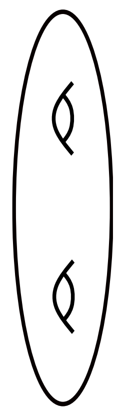
Convex Cocompact $\Longrightarrow$ finitely generated + purely pA

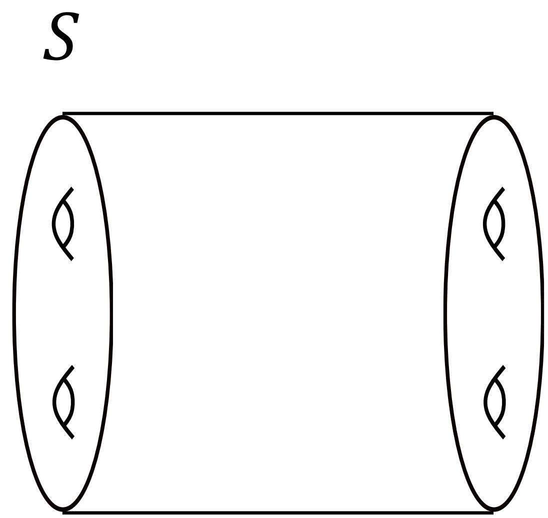
Farb–Mosher: Does fin. gen. + purely pA $\Longrightarrow$ convex cocompact?

Leininger–R. / Dowdall–Kent–Leininger:

Yes, for subgroups of fibered 3-manifold groups.

S

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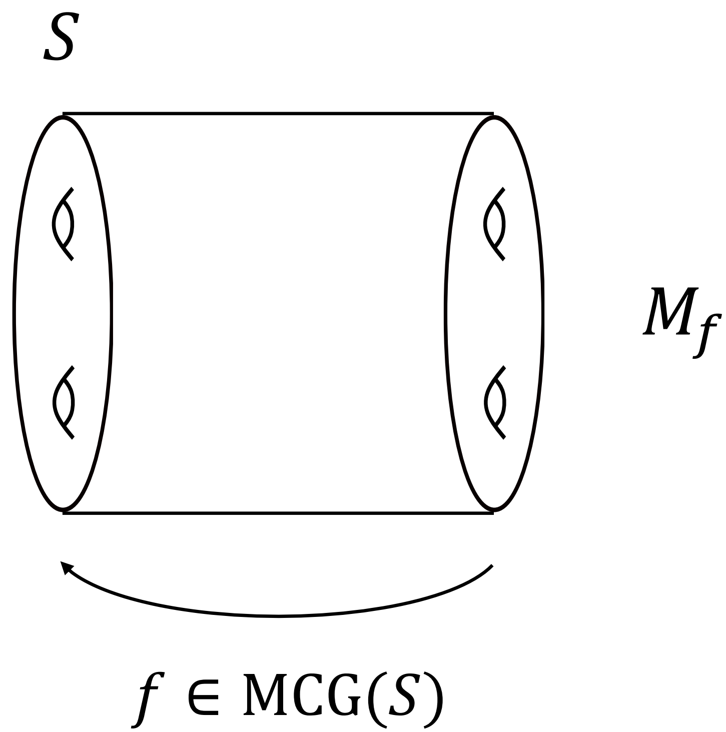
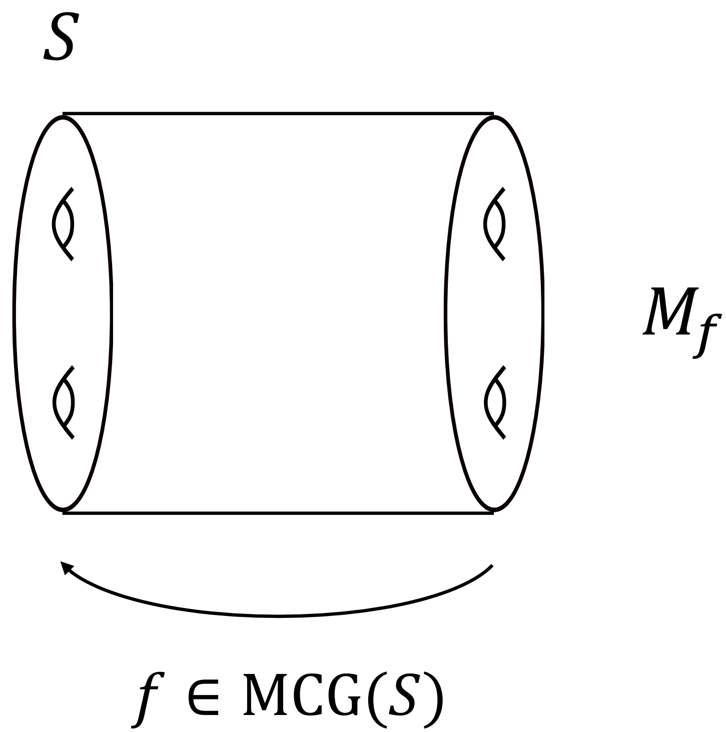
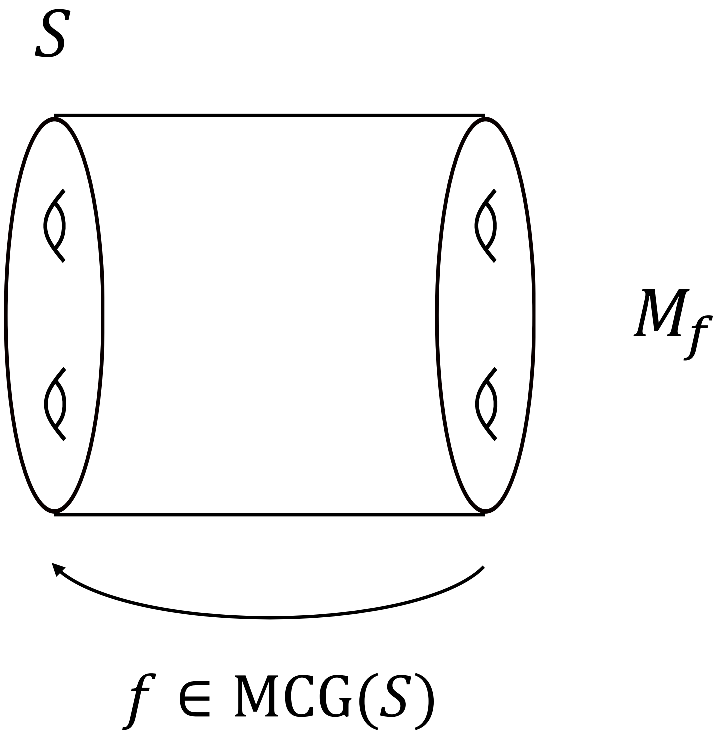

S
M_f
$f \in \mathrm{MCG}(S)$



$\mathrm{MCG}(S)$

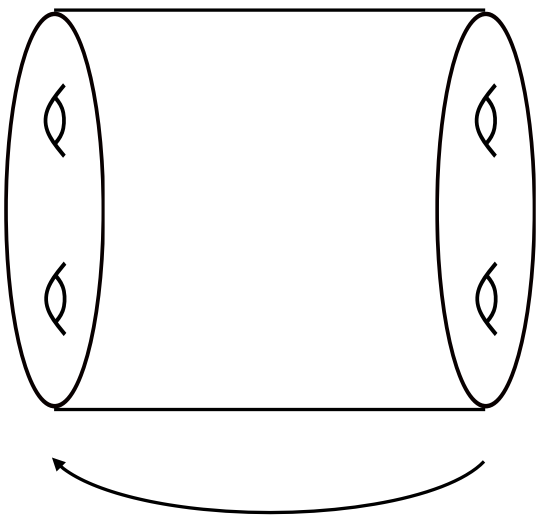
$\vee$

$\langle f \rangle$



$$
\begin{array}{ccccccccc}
1 & \longrightarrow & \pi_1(S) & \longrightarrow & \mathrm{MCG}(S^\circ) & \longrightarrow & \mathrm{MCG}(S) & \longrightarrow & 1 \\
 & & \parallel & & \vee & & \vee & & \\
 & & \pi_1(S) & \longrightarrow & E & \longrightarrow & \langle f \rangle & &
\end{array}
$$

S



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 & & & \text{f ∞-order} & \cong & & & & \\
 & & & & \pi_1(M_f) & & & &
\end{array}
$$

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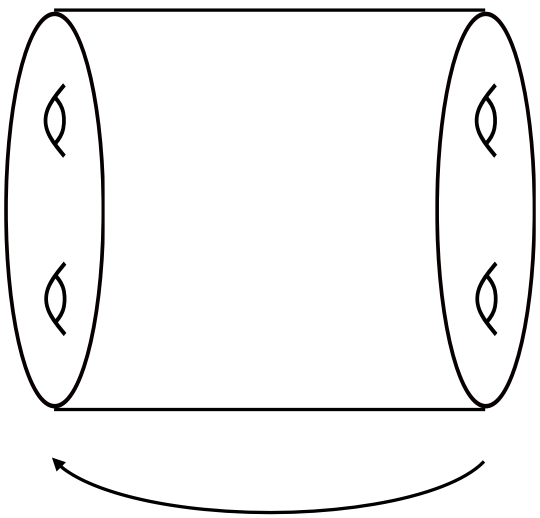

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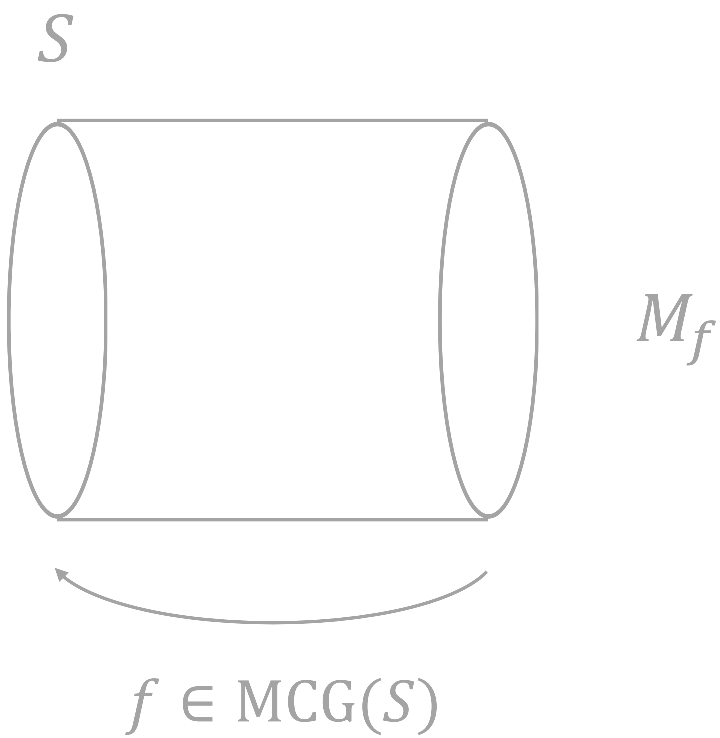
 & & & & \pi_1(M_f) & & & & \\
 & & & & \vee & & & & \\
 & & & & \Gamma & & & & \\
 & & & & \text{f.g. + purely pA} & & & &
\end{array}
$$



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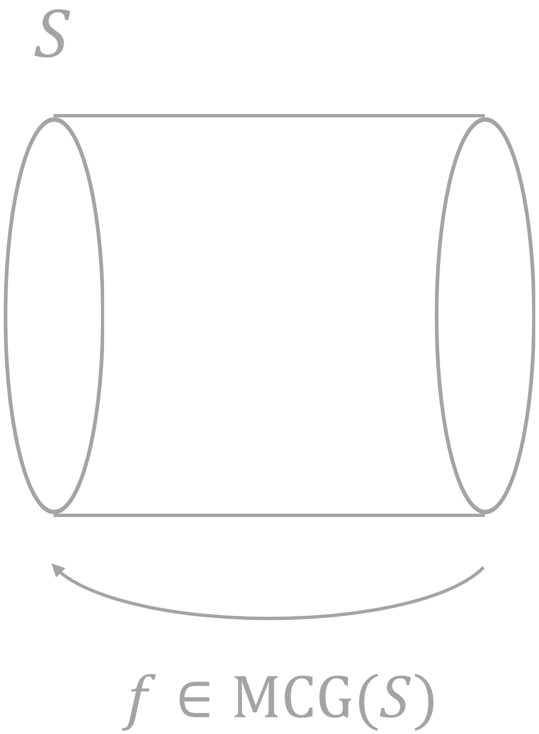
$$

<u>Theorem</u> (Dowdall–Kent–Leininger + Leininger–R.)

Γ fin. gen. + purely pA in $\mathrm{MCG}(S^\circ) \Rightarrow \Gamma$ convex cocompact



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$$

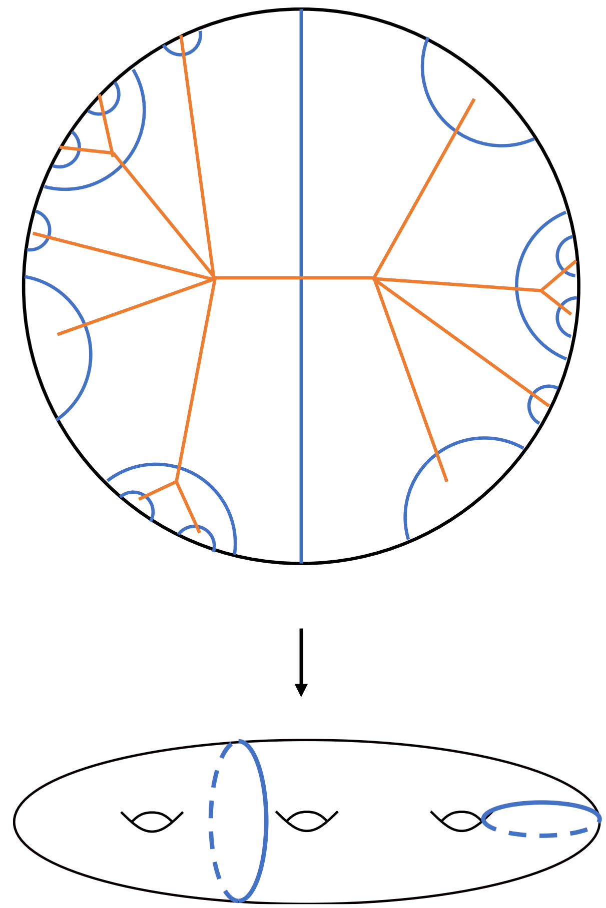
M_f hyp M_f non-hyp

<u>Theorem</u> (Dowdall–Kent–Leininger + Leininger–R.)

Γ fin. gen. + purely pA in $\mathrm{MCG}(S^\circ) \Rightarrow \Gamma$ convex cocompact

Theorem (Leininger –R.)
When M_f is not hyperbolic,

$$\Gamma < \pi_1(M_f) \text{ f.g.} + \text{purely pA in } \mathrm{MCG}(S^\circ)$$
$$\Downarrow$$
$$\Gamma \text{ convex cocompact}$$



A New Condition on the Jones Polynomial of a Fibered Positive Link

Lizzie Buchanan

Dartmouth College

December 9, 2022

The End

Theorem (B., 2022)

The Jones polynomial V_K of a fibered positive knot K satisfies

$$\max\deg V_K \leq 4 \min\deg V_K.$$

Positivity Classification: Is $12_{n148}!$ positive?

If this knot is positive,

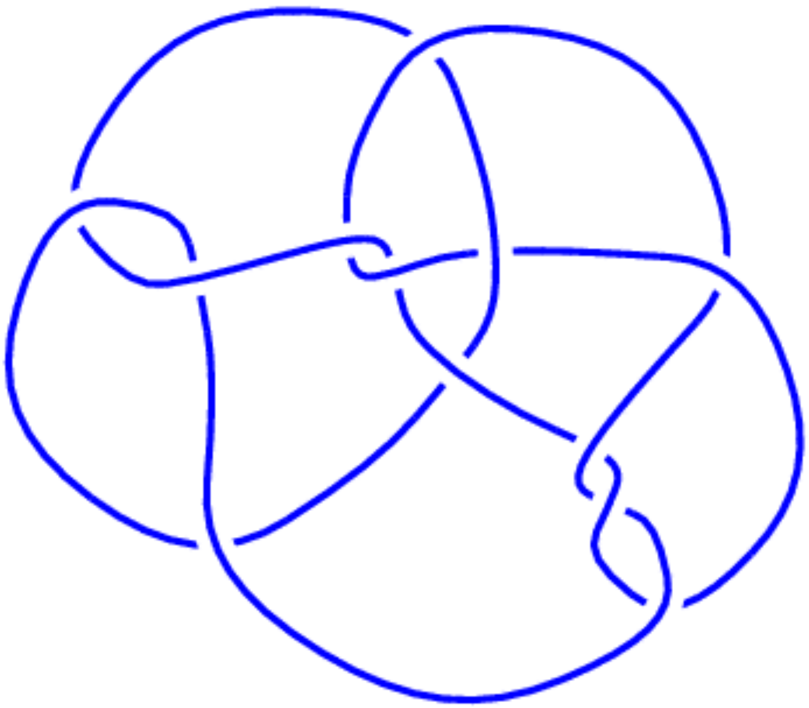
↓ *Stoimenow*

then it is positive and fibered,

↓ *Buchanan*

and so $\max \deg_{V_K} \leq 4 \min \deg_{V_K}$.

Image from KnotInfo



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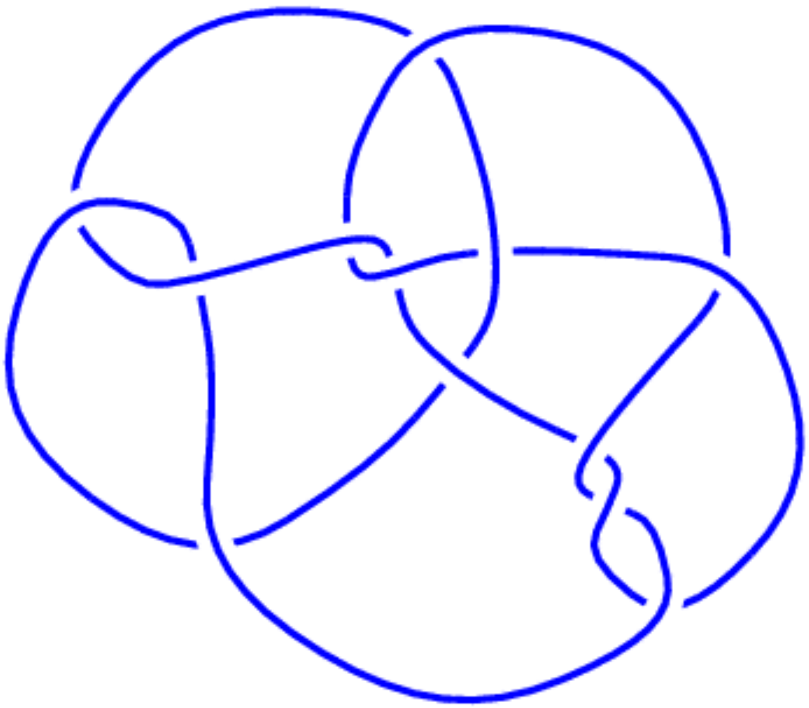

But, Jones polynomial of $12_{n148}!$ is:

$$t^3 + t^6 - 2t^7 + 3t^8 - 3t^9 + 3t^{10} - 3t^{11} + 2t^{12} - t^{13}$$

Positivity Classification: Is $12_{n148}!$ positive?

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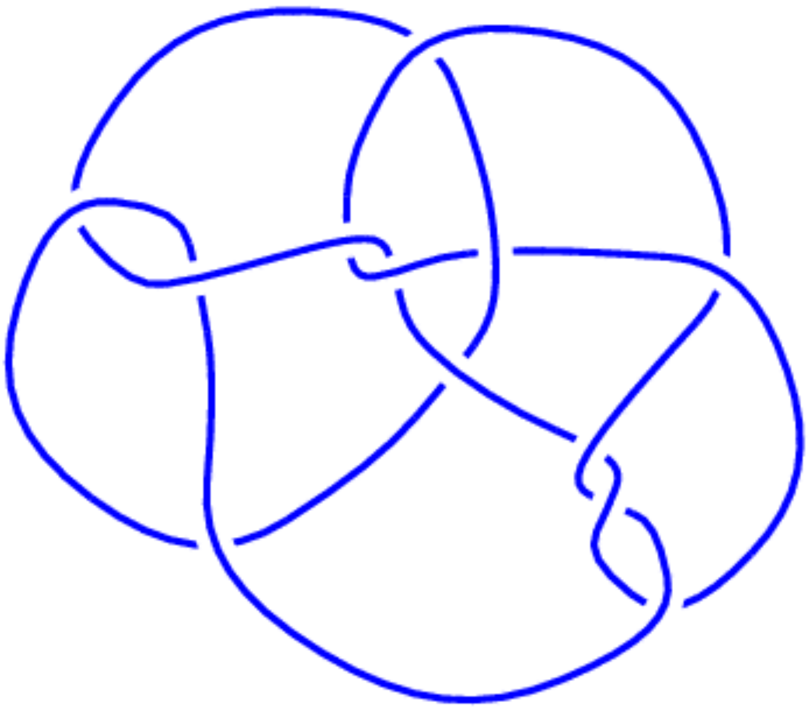

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But, Jones polynomial of $12_{n148}!$ is:

$$t^3 + t^6 - 2t^7 + 3t^8 - 3t^9 + 3t^{10} - 3t^{11} + 2t^{12} - t^{13}$$

with $\max\deg_{V_K} \not\leq 4\min\deg_{V_K}$, and therefore our knot isn't positive.

Positivity classification

All seven of the 12-crossing mystery knots are not positive.

Smoothings

Crossing | *A*-smoothing | *B*-smoothing

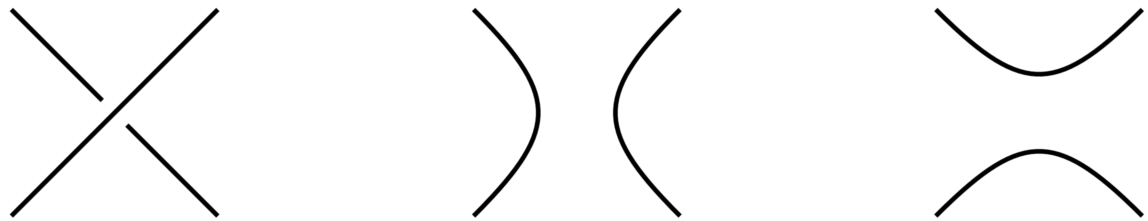
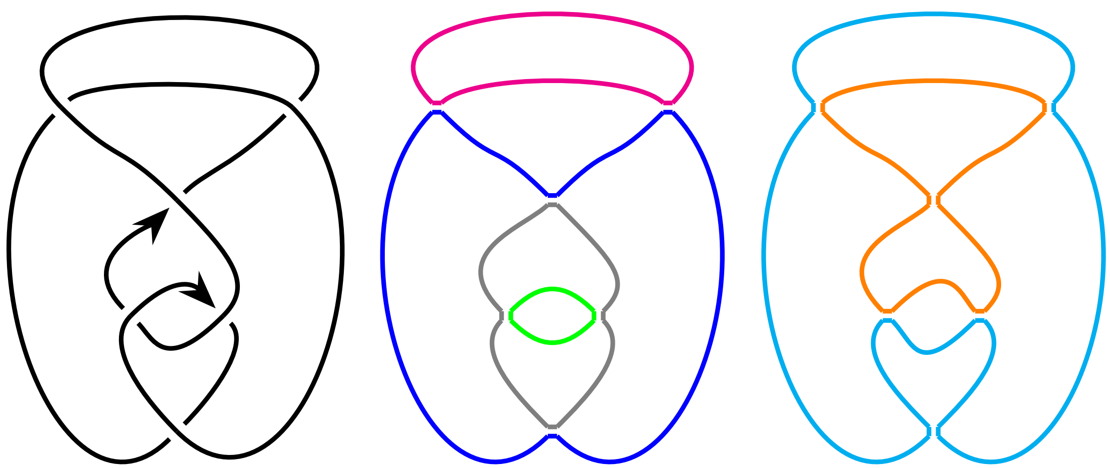


Diagram | *A*-state | *B*-state



Natural bound for positive diagrams

For a positive diagram D, Kauffman state-sum model of Jones polynomial tells us:

$$\max \deg_{V_D} \leq \begin{pmatrix} \text{crossing} \\ \text{number of } D \end{pmatrix} + \frac{\begin{pmatrix} \text{number of circles} \\ \text{in } B\text{-state of } D \end{pmatrix} - 1}{2}$$

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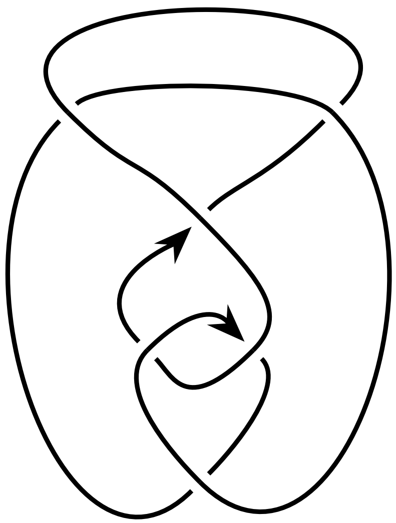
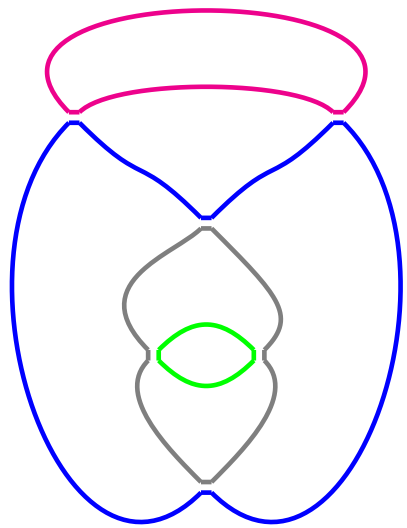
We want to replace these diagram dependent quantities with something that is diagram independent

A-State Graphs

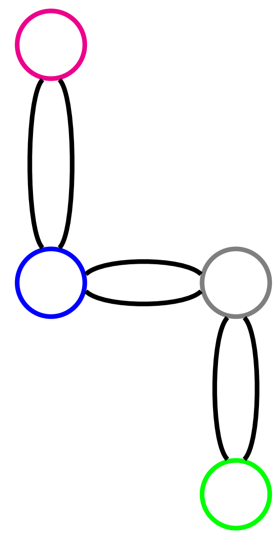
Diagram



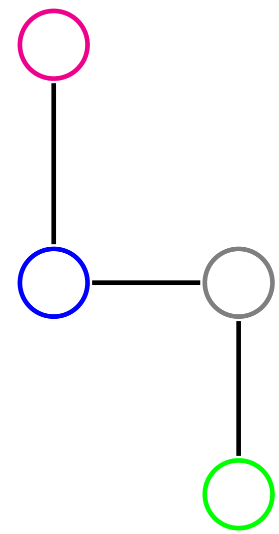
A-state



A-state graph

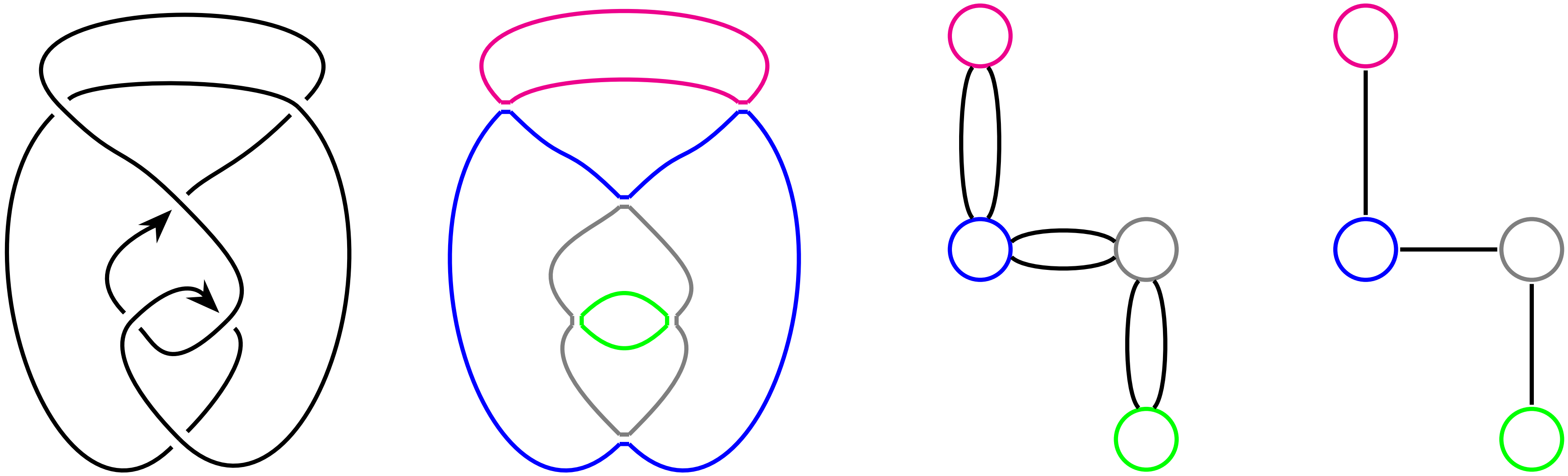


Reduced *A*-state graph



Balanced Diagrams

Diagram | *A*-state | *A*-state graph | Reduced *A*-state graph



Balanced Diagram (roughly): Link diagram whose reduced A-state graph is a tree, and all edges in A-state graph come in pairs

Key Theorem

Theorem (B., 2022)

In a Balanced diagram D, the number of circles in the B-state is equal to the number of link components .

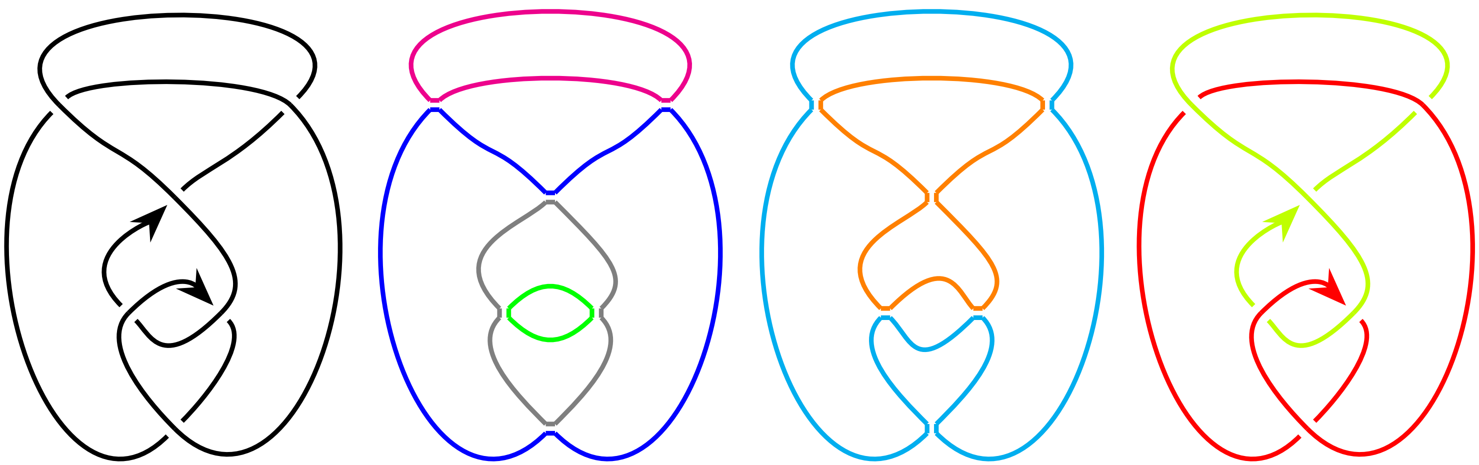
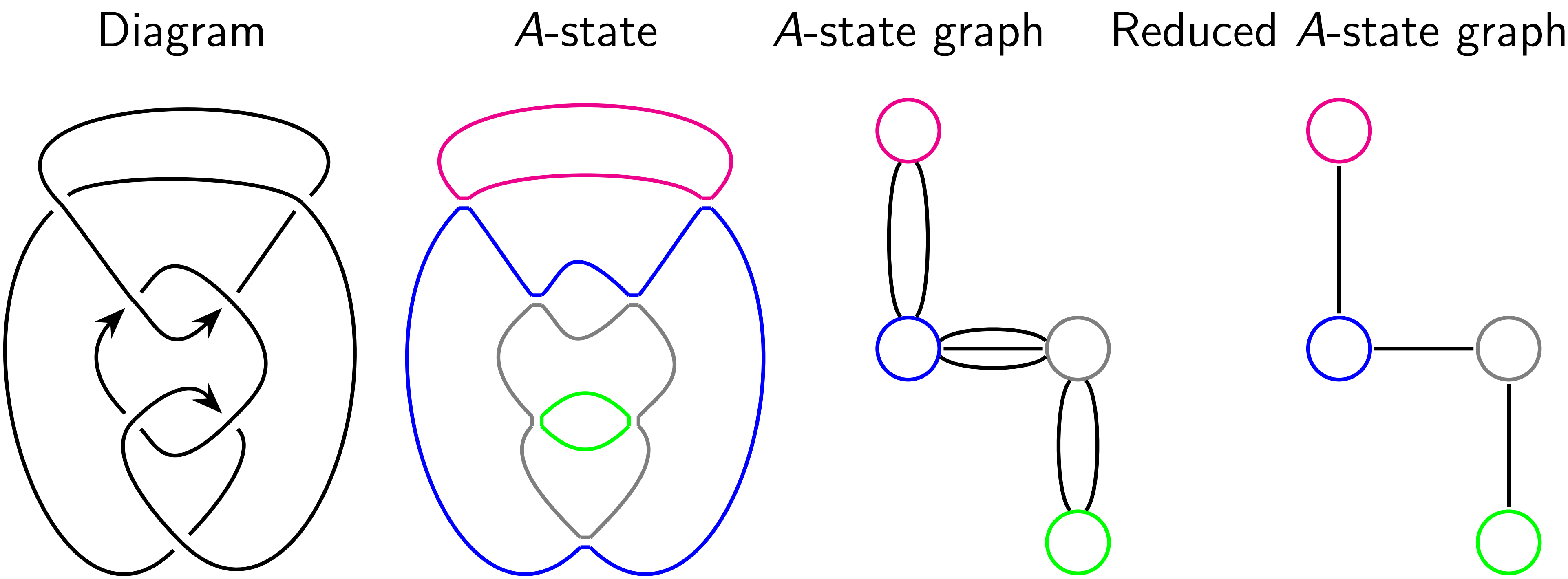


Figure: (Left to right:) A Balanced diagram, its A-state, its B-state, and its components

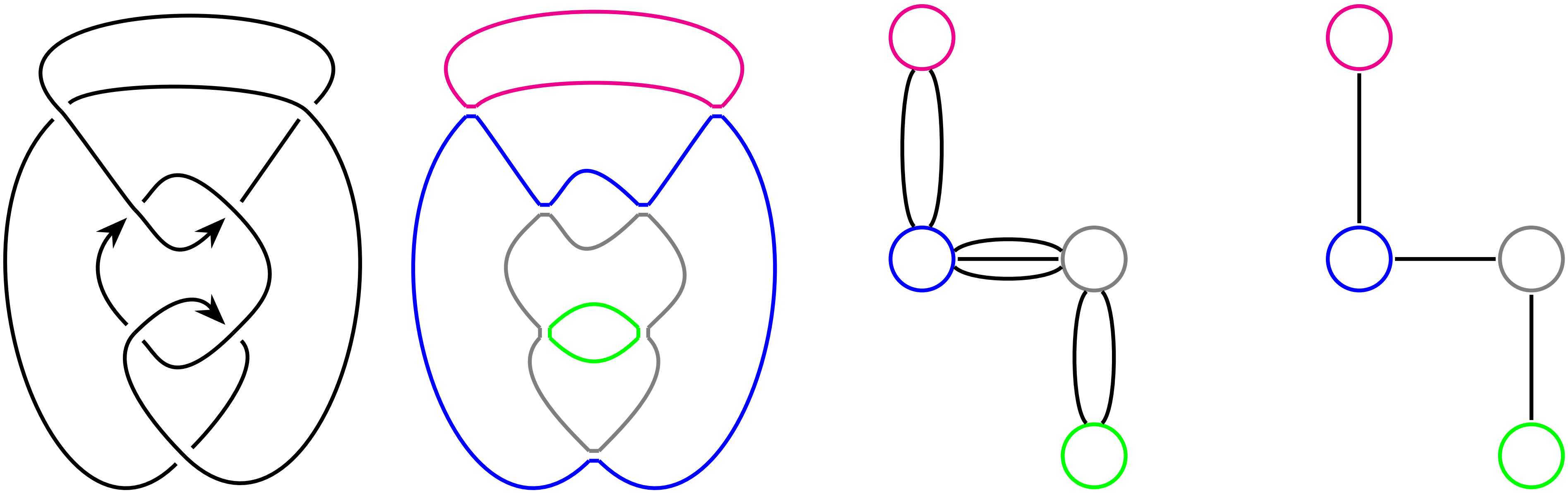
Burdened Diagram



Burdened Diagram (roughly): Underlying reduced A-state graph is a tree, can smooth crossings away to obtain a Balanced diagram

Burdened Diagram

Diagram | *A*-state | *A*-state graph | Reduced *A*-state graph



Burdened Diagram (roughly): Underlying reduced A-state graph is a tree, can smooth crossings away to obtain a Balanced diagram

EVERY reduced positive diagram of a fibered positive link is a Burdened diagram

The End

Theorem (B., 2022)

The Jones polynomial V_L of a fibered positive link with n link components satisfies

$$\max\deg V_L \leq 4\min\deg V_L + \frac{n-1}{2}.$$

In particular, the Jones polynomial V_K of a fibered positive knot K satisfies

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The End

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Thank you!

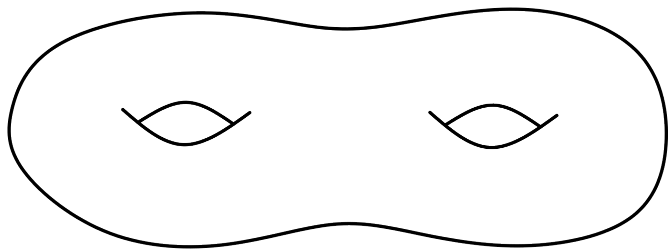
Higher Complex Structures and $\mathrm{SL}(3, \mathbb{R})$ Hitchin Components

Alex Nolte
Rice University

This material is based upon work supported by the National Science Foundation under Grant No. 1842494 and Grant No. 2005551. Thanks to Katherine Booth for illustrations.

Teichmüller Space

S:



$\mathcal{T}(S)$: $\{$complex structures on S$\}/\mathrm{Diff}_0(S)$

$\cong\{$constant curvature -1 metrics on $S\}/\mathrm{Diff}_0(S)$

$\cong\{$discrete, faithful $\rho : \pi_1(S) \to \mathrm{PSL}(2,\mathbb{R})\}/$conjugation

(PSL($n, \mathbb{R}$)) Hitchin Components

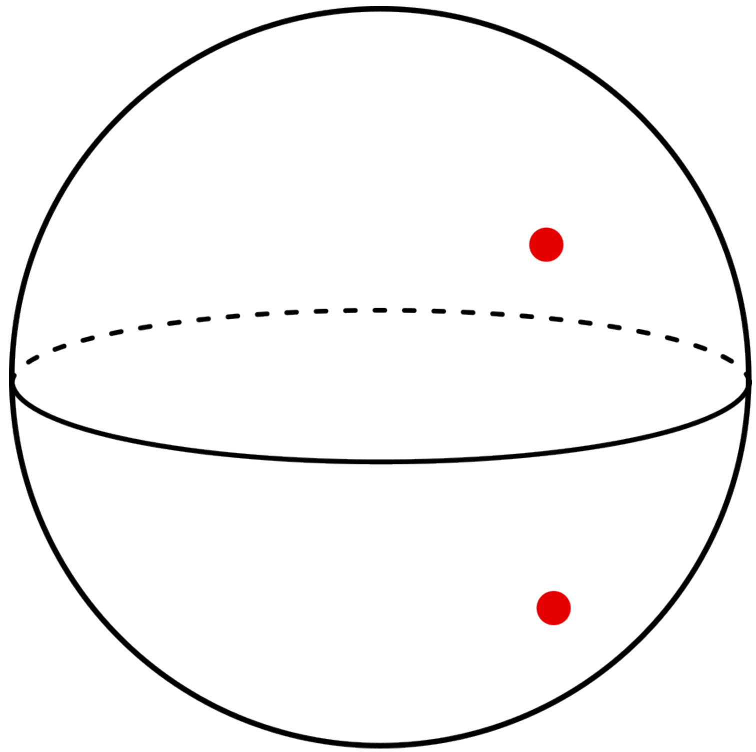
$\mathrm{Hit}_n(S)$:

- Special component of $\mathrm{Hom}(\pi_1(S), \mathrm{PSL}(n, \mathbb{R}))/\mathrm{PSL}(n, \mathbb{R})$
- Remarkable properties analogous to $\mathcal{T}(S)$

Question

What geometric content does $\rho \in \mathrm{Hit}_n(S)$ have?

Rephrasing Almost Complex Structures

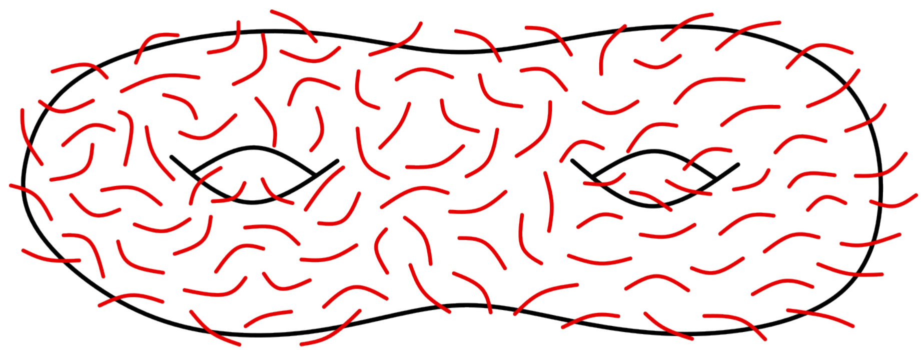


$J_x \in \mathrm{End}(T_x^*S)$ with $J_x^2 = -\mathrm{Id}$ is determined by either:

- $+i$ eigenspace V_x^i,
- Polynomials I_x on $T_x^{*\mathbb{C}}S$ vanishing on V_x^i

Higher (Degree) Complex Structures

- *n*-complex structure I:
  - for every point in x, a special ideal I_x of codimension n in polynomials on $T_x^{*\mathbb{C}}S$.



- Moduli space:
  - $\mathcal{T}^n(S)$: $\{n$-complex structures on S$\}/\mathrm{Ham}_c^0(T^*S)$

Conjecture (Fock-Thomas '18)

There is a natural diffeomorphism $\mathcal{T}^n(S) \cong \mathrm{Hit}_n(S)$

Results (N. '22)

- New realization of n-complex structures
- Basic structure of $\mathcal{T}^n(S)$:
  - Manifold structure
  - $\mathcal{T}^n(S) \cong \mathbb{R}^{-\chi(S)\dim(\mathrm{PSL}(n,\mathbb{R}))}$
  - Complex structure, Kähler metrics
  - Holomorphic vector bundle over $\mathcal{T}(S)$
  - Structure of the $\mathrm{Mod}(S)$-action
- Natural diffeomorphism $\mathcal{T}^3(S) \cong \mathrm{Hit}_3(S)$

Extending Group Actions on Metric Spaces

Joshua B. Perlmutter

Brandeis University

December 9, 2022

Extending Group Actions

G a group, $H \leq G$. Suppose H acts on a metric space R.

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Is there an action of G on a (possibly different) metric space which "extends" $H \curvearrowright R$?

Extending Group Actions

G a group, $H \leq G$. Suppose H acts on a metric space R.

Is there an action of G on a (possibly different) metric space which "extends" $H \curvearrowright R$?

Answer: Sometimes yes, sometimes no.

Goal: Understand the sufficient conditions for actions to extend.

Formalizing the Problem

Definition (Abbott–Hume–Osin)

Let H be a subgroup of G and let $H \curvearrowright R$ be an action of H on a metric space R. An action $G \curvearrowright S$ of G on a metric space S is an *extension* of R if there exists a coarsely H-equivariant quasi-isometric embedding $R \to S$.

Example: Extension Exists

Let $H \leq G$ be a finite subgroup.

Consider any action of H on any metric space R.

The trivial action of G on R is an extension of $H \curvearrowright R$.

Example: No Extension

Consider the action $\mathbb{Z} \curvearrowright \mathbb{R}$ given by $x \cdot r = x + r$.

$\mathbb{Z}$ is countable, so it is a subgroup of $Sym(\mathbb{N})$, the group of all permutations of natural numbers.

Every action of $Sym(\mathbb{N})$ on a metric space S has bounded orbits (Cornulier 2006).

There does not exist a coarsely $\mathbb{Z}$-equivariant map $f : \mathbb{R} \to S$ because $\mathbb{Z} \curvearrowright \mathbb{R}$ is unbounded.

$\mathbb{Z} \curvearrowright \mathbb{R}$ does not extend to an action of $Sym(\mathbb{N})$.

New Result

Goal is to focus on specific actions, rather than subgroup as a whole.

Theorem (P.)

Let G be a group generated by a subset X relative to a subgroup H, and let H act on a metric space R. Let $\Gamma(G, X \sqcup H)$ be hyperbolic. Suppose for some $r_0 \in R$ there exists some constant $C > 0$ such that $\forall h \in H$,

$$d_R(r_0, hr_0) \leq C\hat{d}(1, h).$$

Then $H \curvearrowright R$ can be extended to an action of G on another metric space.

$\hat{d}$ is the relative metric on $\Gamma(G, X \sqcup H)$.

New Result

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$\hat{d}$ is the relative metric on $\Gamma(G, X \sqcup H)$.

New Goal: Find subgroups which admit actions with this condition.

Thank You!

Diagrammatic Presentations of Index 4 Subfactor Planar Algebras

Melody Molander

Tech Topology Conference - GA Tech

December 2022

Thank you organizers!

What is a planar algebra?

The planar algebra TL contains the algebras $TL_k, k \geq 0$, over $\mathbb{C}$.

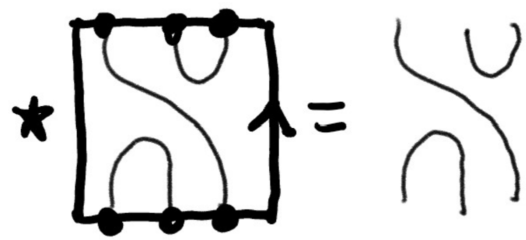
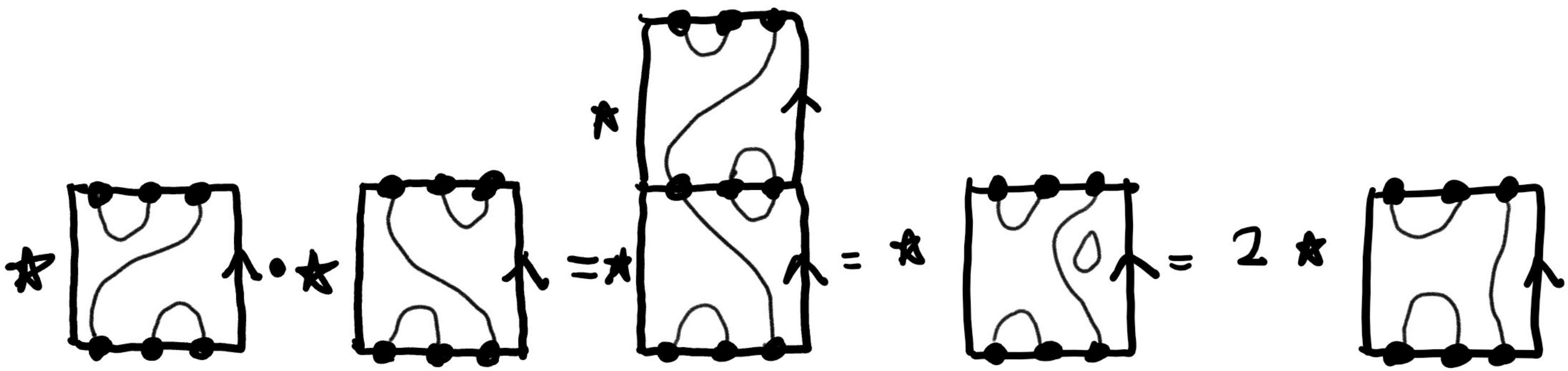


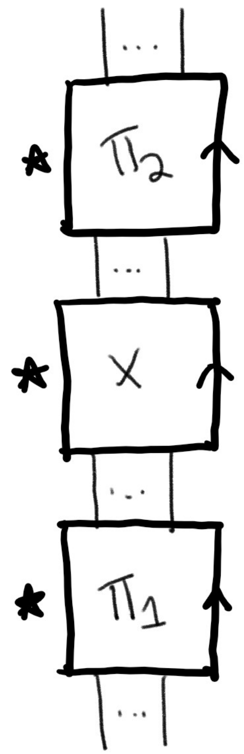
Figure: An element of TL_3

index is 4



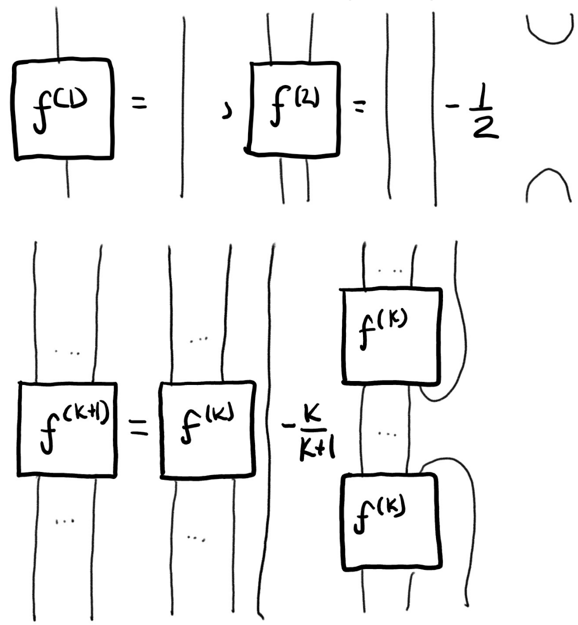
Going from a planar algebra to a tensor category

- Objects: Formal direct sums of projections (i.e., $\pi \in \mathsf{TL}_n$ such that $\pi^2 = \pi$ and $\pi^* = \pi$)
- Morphisms: For π_1, π_2 projections, $\mathrm{Hom}(\pi_1, \pi_2)$ contains diagrams:

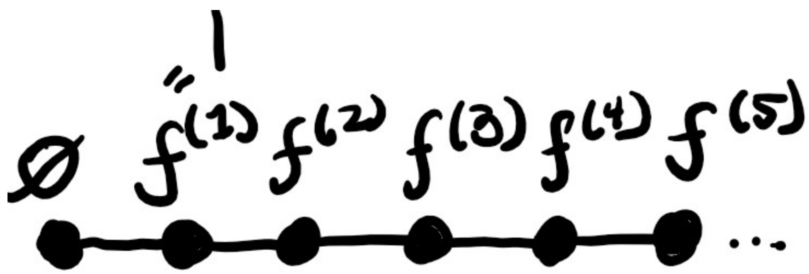


- There's a notion of $\oplus$ and $\otimes$.

- The **Jones-Wenzl projections** $f^{(k)} \in \mathrm{TL}_k$ are the minimal projections in TL defined recursively by:

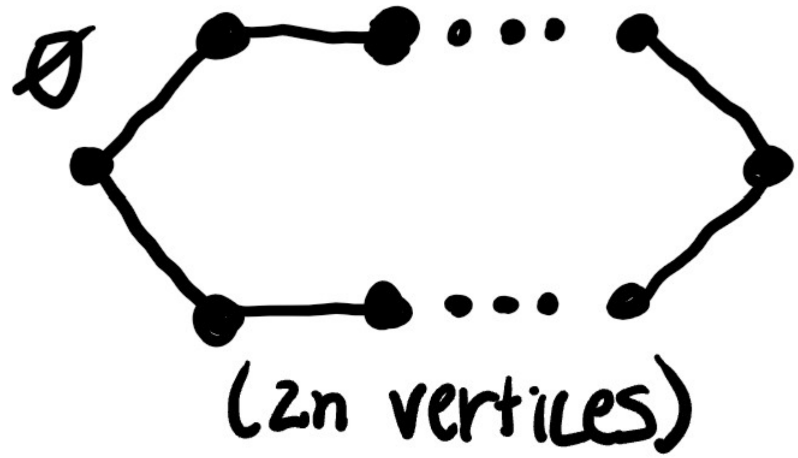


- **Wenzl's relation**: $f^{(k)} \otimes | \cong f^{(k+1)} \oplus f^{(k-1)}$
- Principal graph:



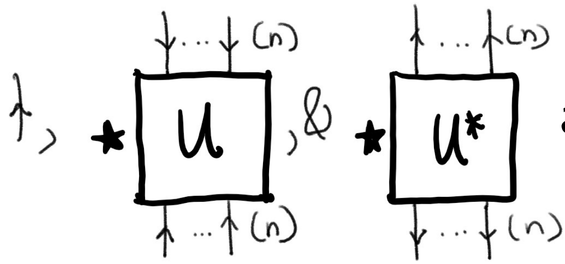
Goal

Find all the subfactor planar algebras of index 4 associated with the $\tilde{A}_{2n-1}$ Dynkin diagram:

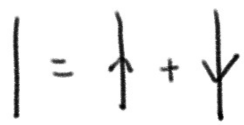


Theorem (M.)

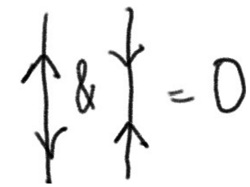
Fix n. Let ω_n be a $2n$th root of unity. Let $\mathcal{PA}(U)$ be the planar algebra with

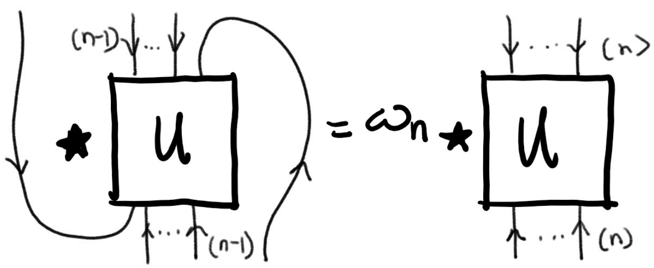
generators:  and relations:

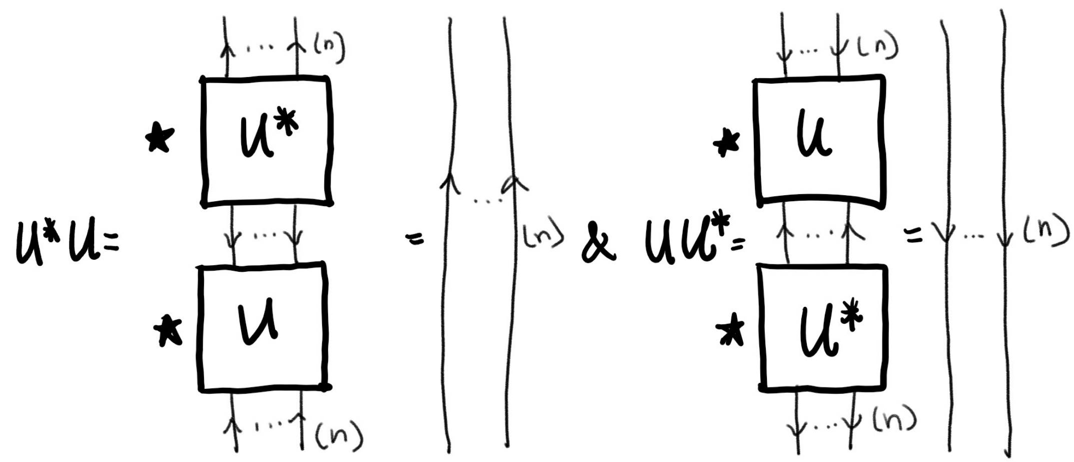
1. 

2. 

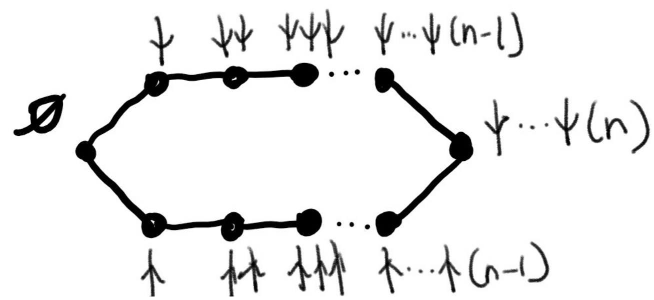
3. 

4. 

5. 

6. 

Then this is an $\tilde{A}_{2n-1}$ subfactor planar algebra of index 4 with principal graph:



Thank you!

Scott Morrison, Emily Peters, and Noah Snyder.
Skein theory for the D_{2n} planar algebras.
J. Pure Appl. Algebra, 214(2):117–139, 2010.