Math 500 - Fall 2001
Midterm

You may use your book and notes on this test. Turn in the exam by noon on Friday November 2.

1) Show a compact metric space is second countable.
Hint: For each positive integer n consider $\{B_{\frac{1}{n}}(x)\}_{x\in X}$. By compactness there is a finite sub cover. Try to assemble these finite sub covers, for various n's, into a countable basis.

2) Let $X = \mathbb{R}^2 \setminus \{(0,0)\}$ and $\mathcal{D} = \{l \cap X | l \text{ is a line in } \mathbb{R}^2 \text{ through the origin}\}$. What is the decomposition space $\mathcal{D}$ homeomorphic to? Prove it.

3) Classify A, B, C, D, E, F, G, H up to homeomorphism. By this I mean break the letters into groups such that letters in the same group are homeomorphic. Then prove that letters in different groups are not homeomorphic. (The letters are all closed subsets of the plane and have the induced topology.)

4) A subset S of a topological space X is called a **zero set** if there is a continuous function $f : X \to [0,1]$ such that $f^{-1}(0) = S$. Prove:

a) zero sets are closed.

b) every closed subset of a metric space is a zero set.

c) if every closed subset of a second countable T_1-space X is a zero set then X is metrizable. (Hint: prove such and X is regular then appeal to a metrization theorem.)

5)Let X be a Hausdorff space and C_i compact connected subspaces such that $C_{i+1} \subset C_i$. Show that

a) $C = \cap_{i=1}^{\infty} C_i$ is non-empty.

b) C is compact.

c) C is connected. (Hint: It will help to think of everything as a subset of C_1 (since you will then be working in a compact space). Assume not and try to prove there are disjoint open sets separating the components of C. Now try to get a contradiction to the fact that the C_i's are connected.)

6) Let U_i be a open dense set in X for each $i = 1 \ldots n$. Prove $\cap_{i=1}^{n} U_i$ is dense.