a reformulation of axiom ac1

Johan G. F. Belinfante
2004 October 30

```
In[1]:= SetDirectory["i:"]; << goedel62.30a; << tools.m;

        :Package Title: goedel62.30a        2004 October 30 at 3:00 p.m.

        It is now:  2004 Oct 30 at 15:12

        Loading Simplification Rules

        TOOLS.M                     Revised 2004 October 28

        weightlimit = 40
```

summary

A reformulation of axiom **ac1** in terms of **RS[inverse[E]]** is derived.

axiom ac1

The axiom **axch = ac1** implies that the restriction of **inverse[E]** to any set has a cross-section. A normalization result is needed:

```
In[2]:= (image[inverse[IMAGE[id[complement[singleton[0]]]]]],
          image[IMAGE[FIRST], intersection[FUNS, P[inverse[E]]]]] //
         Normality // Reverse) /. Equal → SetDelayed
```

The following conditional rewrite rule serves to normalize **RS[inverse[E]]**, and other expressions of a similar nature.

```
In[4]:= SubstTest[implies, equal[y, thinpart[x]], equal[RS[x],
          intersection[invar[composite[id[x], inverse[FIRST], FIRST]], P[y]]]], y → x]

Out[4]= or[equal[
           intersection[invar[composite[id[x], inverse[FIRST], FIRST]], P[x]], RS[x]],
          not[equal[V, domain[VERTSECT[x]]]], not[subclass[x, cart[V, V]]]] == True

In[5]:= intersection[invar[composite[id[x_], inverse[FIRST], FIRST]], P[x_]] :=
         RS[x] /; thin[x] && composite[Id, x] == x
```

Main result: A reformulation of axiom **ac1**.

```
In[6]:= Map[assert[equal[V, #]] &,
         image[inverse[IMAGE[composite[id[inverse[E]], inverse[FIRST]]]], fix[
            composite[S, id[FUNS], inverse[IMAGE[FIRST]], IMAGE[FIRST]]]]] // Normality]

Out[6]= subclass[RS[inverse[E]],
          fix[composite[S, id[FUNS], inverse[IMAGE[FIRST]], IMAGE[FIRST]]]] == axch

In[7]:= subclass[RS[inverse[E]],
          fix[composite[S, id[FUNS], inverse[IMAGE[FIRST]], IMAGE[FIRST]]]] := axch
```

Comment. The class **RS[inverse[E]]** is the class of all (small) restrictions of the inverse membership relation. The statement says that the set version of the axiom of choice is equivalent to the statement that this class is contained in the class of all sets with nonempty **X[x]**.