# cross-sections as maximal functions

Johan G. F. Belinfante
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```
In[1]:= SetDirectory["l:"]; << goedel.08jan12a; << tools.m

        :Package Title: goedel.08jan12a           2008 January 12 at 5:30 p.m.

        It is now:  2008 Jan 13 at 10:1

        Loading Simplification Rules

        TOOLS.M                                   Revised 2008 January 2

        weightlimit = 40
```

## summary

The main idea used in the proof that Zorn's lemma implies the axiom of choice is that cross-sections are maximal functions contained in a power set:

```
In[2]:= class[x, member[intersection[FUNS, P[x]], domain[MAXIMAL[S]]]]

Out[2]= SELECT
```

In this notebook, this observation is used to derive a simple statement equivalent to the axiom of choice which formally resembles a weak version of Zorn's lemma.

## derivation

The following observation led to the discovery of the main theorem derived in this notebook.

```
In[3]:= class[x, member[intersection[t, P[x]], z]] /. {t → FUNS, z → domain[MAXIMAL[S]]}

Out[3]= complement[image[inverse[POWER], image[inverse[IMAGE[id[FUNS]]], subvar[inverse[PS]]]]]
```

Lemma.

```
In[4]:= image[inverse[POWER], image[inverse[IMAGE[id[FUNS]]], subvar[inverse[PS]]]] // Normality

Out[4]= image[inverse[POWER], image[inverse[IMAGE[id[FUNS]]], subvar[inverse[PS]]]] ==
         complement[SELECT]

In[5]:= image[inverse[POWER], image[inverse[IMAGE[id[FUNS]]], subvar[inverse[PS]]]] :=
         complement[SELECT]
```

Main Theorem.

```
In[6]:= SubstTest[empty, image[inverse[POWER], t],
          t -> image[inverse[IMAGE[id[FUNS]]], subvar[inverse[PS]]]]

Out[6]= equal[0, intersection[image[IMAGE[id[FUNS]], range[POWER]], subvar[inverse[PS]]]] == axch

In[7]:= equal[0,
          intersection[image[IMAGE[id[FUNS]], range[POWER]], subvar[inverse[PS]]]] := axch
```

## comments

The theorem derived in the preceding section formally resembles the following weak version of Zorn's lemma:

```
In[8]:= equal[0, intersection[fix[UCHAINS], subvar[inverse[PS]]]]

Out[8]= axch
```

The following connection between these statements could have been used to derive half of the main theorem:

```
In[9]:= Map[equal[V, #] &,
          image[inverse[POWER], image[inverse[IMAGE[id[FUNS]]], fix[UCHAINS]]] // Normality]

Out[9]= subclass[image[IMAGE[id[FUNS]], range[POWER]], fix[UCHAINS]] == True

In[10]:= subclass[image[IMAGE[id[FUNS]], range[POWER]], fix[UCHAINS]] := True
```