

a conditional fixed point rule

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```
In[1]:= (* SetDirectory["l:"]; *) << goedel.08jun17a; << tools.m
:Package Title: goedel.08jun17a                2008 June 17 at 1:00 p.m.
It is now: 2008 Jun 18 at 9:34
Loading Simplification Rules
TOOLS.M                                       Revised 2008 May 17
weightlimit = 40
```

summary

A new conditional rewrite rule is derived which is analogous to an existing one.

derivation

Lemma.

```
In[7]:= SubstTest[implies, subclass[x, t],
               equal[fix[composite[x, inverse[t]]], range[x]], t → inverse[y]] // Reverse
Out[7]:= or[equal[fix[composite[x, y]], range[x]], not[subclass[x, inverse[y]]]] == True
In[8]:= or[equal[fix[composite[x_, y_]], range[x_]], not[subclass[x_, inverse[y_]]]] := True
```

Theorem. (A new conditional rewrite rule.)

```
In[9]:= implies[subclass[x, inverse[y]], equal[fix[composite[x, y]], range[x]]]
Out[9]:= True
In[11]:= fix[composite[x_, y_]] := range[x] /; simplify && subclass[x, inverse[y]]
```

an unconditional rule

The simplify flag is turned off in this section.

```
In[12]:= simplify = False;
```

An unconditional rewrite rule analogous to the following existing rule will now be derived.

```
In[15]:= fix[composite[wf[x], Di]]
Out[15]:= range[wf[x]]
```

Theorem.

```
In[13]= SubstTest[union, fix[t], fix[composite[Di, t]], t → wf[x]] // Reverse
```

```
Out[13]= fix[composite[Di, wf[x]]] == domain[wf[x]]
```

```
In[14]= fix[composite[Di, wf[x_]]] := domain[wf[x]]
```