inequalities for natdiv

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```
In[1]:= SetDirectory["l:"]; << goedel91.11a; << tools.m

        :Package Title: goedel91.11a        2007 March 11 at 11:20 a.m.

        It is now:  2007 Mar 13 at 3:34

        Loading Simplification Rules

        TOOLS.M                       Revised 2007 March 3

        weightlimit = 40
```

summary

A conditional rewrite rule is derived for the literal **member[natdiv[x, y], z]** which rewrites such literals, replacing division with multiplication under the condition that **z** be a natural number.

preliminaries

The results in this section hold independently of any condition on **z**.

```
In[2]:= SubstTest[implies, member[u, z], member[u, V], u → natdiv[x, y]] // Reverse

Out[2]= or[member[pair[y, x], DIV], not[member[natdiv[x, y], z]]] == True

In[3]:= or[member[pair[y_, x_], DIV], not[member[natdiv[x_, y_], z_]]] := True
```

Corollaries.

```
In[4]:= Map[not,
          SubstTest[and, implies[p1, p2], not[implies[p1, p3]], {p1 -> member[natdiv[x, y], z],
            p2 -> member[pair[y, x], DIV], p3 → member[x, omega]}]] // Reverse

Out[4]= or[member[x, omega], not[member[natdiv[x, y], z]]] == True

In[5]:= or[member[x_, omega], not[member[natdiv[x_, y_], z_]]] := True

In[6]:= Map[not,
          SubstTest[and, implies[p1, p2], not[implies[p1, p3]], {p1 -> member[natdiv[x, y], z],
            p2 -> member[pair[y, x], DIV], p3 → member[y, omega]}]] // Reverse

Out[6]= or[member[y, omega], not[member[natdiv[x, y], z]]] == True
```

```
In[7]:= or[member[y_, omega], not[member[natdiv[x_, y_], z_]]] := True
```

derivation

Lemma.

```
In[8]:= Map[or[equal[0, y], member[x, natmul[y, z]],
            not[member[z, omega]], not[member[natdiv[x, y], z]]] &,
          SubstTest[member, natmul[t, y], natmul[z, y], t → natdiv[x, y]] // MapNotNot] // Reverse
```

```
Out[8]= or[equal[0, y], member[x, natmul[y, z]],
          not[member[z, omega]], not[member[natdiv[x, y], z]]] == True
```

```
In[9]:= or[equal[0, y_], member[x_, natmul[y_, z_]],
          not[member[z_, omega]], not[member[natdiv[x_, y_], z_]]] := True
```

Lemma.

```
In[10]:= Map[or[member[natdiv[x, y], z], not[member[x, natmul[y, z]]],
            not[member[z, omega]], not[member[pair[y, x], DIV]], #] &,
          SubstTest[member, natmul[t, y], natmul[z, y], t → natdiv[x, y]] // MapNotNot]
```

```
Out[10]= or[member[natdiv[x, y], z], not[member[x, natmul[y, z]]],
           not[member[z, omega]], not[member[pair[y, x], DIV]]] == True
```

```
In[11]:= or[member[natdiv[x_, y_], z_], not[member[x_, natmul[y_, z_]]],
           not[member[z_, omega]], not[member[pair[y_, x_], DIV]]] := True
```

Main theorem.

```
In[12]:= implies[member[z, omega], equiv[member[natdiv[x, y], z],
             or[and[member[x, natmul[y, z]], member[pair[y, x], DIV]],
              and[equal[0, x], equal[0, y], not[equal[0, z]]]]]] // not // not
```

```
Out[12]= True
```

```
In[13]:= member[natdiv[x_, y_], z_] := or[and[member[x, natmul[y, z]], member[pair[y, x], DIV]],
            and[equal[0, x], equal[0, y], not[equal[0, z]]]] /; member[z, omega]
```