

Quaife's Theorem (DV15)

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```
In[1]:= SetDirectory["1:"]; << goedel92.10a; << tools.m

:Package Title: goedel92.10a      2007 April 10 at 11:20 a.m.

It is now: 2007 Apr 10 at 16:28

Loading Simplification Rules

TOOLS.M                          Revised 2007 March 25

weightlimit = 40
```

summary

In this notebook the third and fourth clauses of Quaife's Theorem (**DV15**) are derived. The fourth clause can be simplified by omitting one literal.

```
In[2]:= "Art Quaife, Automated Development of Fundamental
        Mathematical Theories, Appendix 3. Theorems Proved in Peano's
        Arithmetic, Kluwer Academic Publishers, Dordrecht, 1992. Cf. p. 199";
```

Quaife's clause (DV15c)

Theorem.

```
In[3]:= Map[not, SubstTest[and, implies[p2, p3], implies[and[p1, p2], p4],
                implies[p4, p5], implies[and[p1, p3, p5], p6], not[implies[and[p1, p2], p6]],
                {p1 → member[pair[x, y], DIV], p2 → member[pair[z, x], DIV],
                p3 → equal[0, natmod[x, z]], p4 → member[pair[z, y], DIV], p5 → equal[0, natmod[y, z]],
                p6 → member[pair[natmod[x, natmod[x, z]], natmod[y, natmod[y, z]]], DIV}}] // Reverse
```

```
Out[3]= or[member[pair[natmod[x, natmod[x, z]], natmod[y, natmod[y, z]]], DIV],
         not[member[pair[x, y], DIV], not[member[pair[z, x], DIV]]] = True
```

```
In[4]:= or[member[pair[natmod[x_, natmod[x_, z_]], natmod[y_, natmod[y_, z_]]], DIV],
         not[member[pair[x_, y_], DIV], not[member[pair[z_, x_], DIV]]] := True
```

This implies clause (**DV15c**).

```
In[5]:= implies[and[member[pair[x, y], DIV], member[pair[z, x], DIV]],
  member[pair[natquot[x, z], natquot[y, z]], DIV]
```

```
Out[5]= True
```

a simplified version of Quaife's clause (DV15d)

Theorem.

```
In[6]:= Map[not, SubstTest[and, implies[p1, p2], implies[p2, p3], implies[and[p1, p3], p4],
  not[implies[p1, p4]], {p1 → member[pair[natmul[x, z], y], DIV],
  p2 → member[pair[x, y], DIV], p3 → equal[0, natmod[y, x]],
  p4 → member[pair[natmul[x, z], natsub[y, natmod[y, x]]], DIV}}] // Reverse
```

```
Out[6]= or[member[pair[natmul[x, z], natsub[y, natmod[y, x]]], DIV],
  not[member[pair[natmul[x, z], y], DIV]] = True
```

```
In[7]:= or[member[pair[natmul[x_, z_], natsub[y_, natmod[y_, x_]]], DIV],
  not[member[pair[natmul[x_, z_], y_], DIV]] := True
```

Quaife's clause (DV15d) contains a redundant literal. The following simplified version has been derived.

```
In[8]:= implies[member[pair[natmul[x, z], y], DIV], member[pair[z, natquot[y, x]], DIV]
```

```
Out[8]= True
```