

membership rules for MINIMAL and MAXIMAL

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```
In[1]:= SetDirectory["1:"]; << goedel.08jun13a; << tools.m

:Package Title: goedel.08jun13a          2008 June 13 at 11:50 p.m.

It is now: 2008 Jun 17 at 12:40

Loading Simplification Rules

TOOLS.M                                Revised 2008 May 17

weightlimit = 40
```

summary

Membership rules for **MINIMAL**[z] and **MAXIMAL**[z] are derived.

derivation

Lemma. (Temporary rewrite rule.)

```
In[2]:= equiv[and[member[x, V], member[y, V], member[pair[x, y], MINIMAL[z]]],
            member[pair[x, y], MINIMAL[z]] // not // not
```

```
Out[2]= True
```

```
In[3]:= and[member[x_, V], member[y_, V], member[pair[x_, y_], MINIMAL[z_]]] :=
        member[pair[x, y], MINIMAL[z]]
```

Theorem. (Membership rule for **MINIMAL**.)

```
In[4]:= SubstTest[member, y, image[t, set[x]], t → MINIMAL[z]]
```

```
Out[4]= member[pair[x, y], MINIMAL[z]] ==
        and[member[x, V], member[y, x], not[member[y, fix[composite[z, id[x], Di]]]]]
```

```
In[5]:= member[pair[x_, y_], MINIMAL[z_]] :=
        and[member[x, V], member[y, x], not[member[y, fix[composite[z, id[x], Di]]]]]
```

Corollary. (Membership rule for **MAXIMAL**.)

```
In[6]:= SubstTest[member, pair[x, y], MINIMAL[t], t → inverse[z]] // Reverse
Out[6]= member[pair[x, y], MAXIMAL[z]] ==
        and[member[x, V], member[y, x], not[member[y, fix[composite[Di, id[x], z]]]]]

In[7]:= member[pair[x_, y_], MAXIMAL[z_]] :=
        and[member[x, V], member[y, x], not[member[y, fix[composite[Di, id[x], z]]]]]
```