Quaife's Theorem (O21)

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summary

The first clause of Quaife's Theorem (O21) is already recognized by the GOEDEL program. New rewrite rules are derived here to take care of the second clause.

derivation

Lemma:

The following justifies a new conditional rewrite rule:
To facilitate computation, the GOEDEL program currently rewrites $2x$ as $x + x$. On account of this, the following rule is needed:

```
In[6] := SubstTest[member, natmul[nat[x], nat[z]],
                     natmul[nat[y], nat[z]], z -> succ[set[0]]]
Out[6] = member[natadd[nat[x], nat[x]], natadd[nat[y], nat[y]]] = member[nat[x], nat[y]]

In[7] := member[natadd[nat[x_], nat[x_]], natadd[nat[y_], nat[y_]]] :=
       member[nat[x], nat[y]]
```

The second clause of (O21) is now recognized:

```
In[8] := member[natadd[set[0], natmul[succ[set[0]], nat[x]]],
             natadd[set[0], natmul[succ[set[0]], nat[y]]]]
Out[8] = member[nat[x], nat[y]]
```