

---

# equations for natdiv

Johan G. F. Belinfante  
2007 March 11

```
In[1]:= SetDirectory["1:"; << goedel91.10a; << tools.m

:Package Title: goedel91.10a          2007 March 10 at 11:05 p.m.

It is now: 2007 Mar 11 at 9:14

Loading Simplification Rules

TOOLS.M          Revised 2007 March 3

weightlimit = 40
```

---

## summary

A rewrite rule is derived that automatically rewrites simple equations involving **natdiv** as equations involving multiplication.

---

## derivation

Theorem.

```
In[3]:= equiv[equal[x, natdiv[y, z]], or[and[equal[v, x], not[member[pair[z, y], DIV]]],
      and[equal[0, x], equal[0, y], equal[0, z]],
      and[equal[0, x], equal[0, y], member[z, omega]], and[equal[y, natmul[x, z]],
      member[x, omega], member[z, omega], not[equal[0, z]]]]] // not // not

Out[3]= True

In[5]:= equal[x_, natdiv[y_, z_]] :=
  or[and[equal[v, x], not[member[pair[z, y], DIV]]], and[equal[0, x],
  equal[0, y], equal[0, z]], and[equal[0, x], equal[0, y], member[z, omega]],
  and[equal[y, natmul[x, z]], member[x, omega], member[z, omega], not[equal[0, z]]]]]
```

---

## examples

An example involving a specific number:

```
In[6]:= equal[natdiv[x, y], succ[set[0]]]

Out[6]= and[equal[x, natadd[y, y]], member[y, omega], not[equal[0, y]]]
```

An example involving **nat** wrappers:

```
In[7]:= equal[nat[x], natdiv[nat[y], nat[z]]]  
Out[7]= or[and[equal[0, nat[x]], equal[0, nat[y]]],  
         and[equal[nat[y], natmul[nat[x], nat[z]]], not[equal[0, nat[z]]]]]
```

A more complicated example: (equality of floored divisions)

```
In[8]:= equal[natdiv[natsub[nat[x], natmod[nat[x], nat[y]]], nat[y]],  
           natdiv[natsub[nat[z], natmod[nat[z], nat[y]]], nat[y]]]  
Out[8]= or[and[member[nat[x], nat[y]], member[nat[z], nat[y]]], equal[0, nat[y]],  
          equal[natsub[nat[x], natmod[nat[x], nat[y]]], natsub[nat[z], natmod[nat[z], nat[y]]]]]
```