

# composites with GREATEST[x] and LEAST[x]

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```
In[1]:= << goedel53.24d; << tools.m

:Package Title: goedel53.24d      2004 January 24 at 10:00 p.m.

It is now: 2004 Jan 26 at 13:18

Loading Simplification Rules

TOOLS.M                          Revised 2004 January 3

weightlimit = 40
```

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## summary

New formulas involving composites with **GREATEST[x]** and **LEAST[x]** are derived.

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## basic formulas

```
In[2]:= composite[GREATEST[x], inverse[LB[y]]] // ReInRenormality
Out[2]= composite[GREATEST[x], inverse[LB[y]]] = composite[id[fix[x]], y]

In[3]:= composite[GREATEST[x_], inverse[LB[y_]]] := composite[id[fix[x]], y]

In[4]:= composite[GREATEST[x], inverse[UB[y]]] // ReInRenormality
Out[4]= composite[GREATEST[x], inverse[UB[y]]] = composite[id[fix[x]], inverse[y]]

In[5]:= composite[GREATEST[x_], inverse[UB[y_]]] := composite[id[fix[x]], inverse[y]]
```

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## inverse formulas

```
In[6]:= composite[LB[x], inverse[GREATEST[y]]] // DoubleInverse
Out[6]= composite[LB[x], inverse[GREATEST[y]]] = composite[inverse[x], id[fix[y]]]

In[7]:= composite[LB[x_], inverse[GREATEST[y_]]] := composite[inverse[x], id[fix[y]]]

In[8]:= composite[UB[x], inverse[GREATEST[y]]] // DoubleInverse
Out[8]= composite[UB[x], inverse[GREATEST[y]]] = composite[x, id[fix[y]]]

In[9]:= composite[UB[x_], inverse[GREATEST[y_]]] := composite[x, id[fix[y]]]
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## formulas for LEAST[x]

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In[10]:= composite[LEAST[x], inverse[LB[y]]] // ReInRenormality
Out[10]= composite[LEAST[x], inverse[LB[y]]] = composite[id[fix[x]], y]

In[11]:= composite[LEAST[x_], inverse[LB[y_]]] := composite[id[fix[x]], y]

In[12]:= composite[LEAST[x], inverse[UB[y]]] // ReInRenormality
Out[12]= composite[LEAST[x], inverse[UB[y]]] = composite[id[fix[x]], inverse[y]]

In[13]:= composite[LEAST[x_], inverse[UB[y_]]] := composite[id[fix[x]], inverse[y]]

```

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## inverse formulas

```

In[14]:= composite[LB[x], inverse[LEAST[y]]] // DoubleInverse
Out[14]= composite[LB[x], inverse[LEAST[y]]] = composite[inverse[x], id[fix[y]]]

In[15]:= composite[LB[x_], inverse[LEAST[y_]]] := composite[inverse[x], id[fix[y]]]

In[16]:= composite[UB[x], inverse[LEAST[y]]] // DoubleInverse
Out[16]= composite[UB[x], inverse[LEAST[y]]] = composite[x, id[fix[y]]]

In[17]:= composite[UB[x_], inverse[LEAST[y_]]] := composite[x, id[fix[y]]]

```

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## formulas involving DISJOINT

The **DISJOINT** formulas could be derived by specializing the above formula, using the following observation:

```

In[18]:= UB[complement[E]]
Out[18]= DISJOINT

```

The results can also be derived directly without knowledge of this fact.

```

In[19]:= composite[GREATEST[x], DISJOINT] // ReInRenormality
Out[19]= composite[GREATEST[x], DISJOINT] = composite[id[fix[x]], complement[inverse[E]]]

In[20]:= composite[GREATEST[x_], DISJOINT] := composite[id[fix[x]], complement[inverse[E]]]

In[21]:= composite[DISJOINT, inverse[GREATEST[x]]] // DoubleInverse
Out[21]= composite[DISJOINT, inverse[GREATEST[x]]] = composite[complement[E], id[fix[x]]]

In[22]:= composite[DISJOINT, inverse[GREATEST[x_]]] := composite[complement[E], id[fix[x]]]

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```
In[23]:= composite[LEAST[x], DISJOINT] // ReInRenormality
Out[23]= composite[LEAST[x], DISJOINT] = composite[id[fix[x]], complement[inverse[E]]]

In[24]:= composite[LEAST[x_], DISJOINT] := composite[id[fix[x]], complement[inverse[E]]]

In[25]:= composite[DISJOINT, inverse[LEAST[x]]] // DoubleInverse
Out[25]= composite[DISJOINT, inverse[LEAST[x]]] = composite[complement[E], id[fix[x]]]

In[26]:= composite[DISJOINT, inverse[LEAST[x_]]] := composite[complement[E], id[fix[x]]]
```