combinatorial intersection of topologies

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```
In[1]:= SetDirectory["l:"]; << goedel.11mar24a

        :Package Title: goedel.11mar24a               2011 March 24 at 10:35 a.m.

        It is now:  2011 Mar 25 at 14:45

        Loading Simplification Rules

        TOOLS.M is now incorporated in the GOEDEL program as of 2010 September 3

        weightlimit = 40
```

summary

If **x** and **y** are topologies, then **Uclosure[image[CAP, cart[x, y]]]** is a topology.

derivation

Lemma.

```
In[12]:= Map[
           implies[and[subclass[image[CAP, cart[x, x]], x], subclass[image[CAP, cart[y, y]], y]],
             subclass[#, image[CAP, cart[x, y]]]] &, ImageComp[
            composite[CAP, cross[CAP, CAP]], TWIST, cart[cart[x, x], cart[y, y]]]] // Reverse

Out[12]= or[not[subclass[image[CAP, cart[x, x]], x]], not[subclass[image[CAP, cart[y, y]], y]],
           subclass[image[CAP, cart[image[CAP, cart[x, y]], image[CAP, cart[x, y]]]],
            image[CAP, cart[x, y]]]] == True

In[14]:= or[not[subclass[image[CAP, cart[x_, x_]], x_]],
           not[subclass[image[CAP, cart[y_, y_]], y_]],
           subclass[image[CAP, cart[image[CAP, cart[x_, y_]], image[CAP, cart[x_, y_]]]],
            image[CAP, cart[x_, y_]]]] := True
```

Theorem.

```
In[17]:= Map[empty[range[complement[#]]] &,
          SubstTest[class, pair[x, y], implies[and[member[x, t], member[y, t]],
           member[image[CAP, cart[x, y]], t]], t → binclosed[CAP]]]

Out[17]= subclass[image[IMAGE[CAP], image[CART, cart[binclosed[CAP], binclosed[CAP]]]],
           binclosed[CAP]] == True
```

```
In[18]:= subclass[image[IMAGE[CAP], image[CART, cart[binclosed[CAP], binclosed[CAP]]]],
           binclosed[CAP]] := True
```

Corollary.

```
In[20]:= SubstTest[implies, and[subclass[u, v], subclass[v, w]], subclass[u, w],
           {u → cart[TOPS, TOPS], v → cart[binclosed[CAP], binclosed[CAP]],
            w → image[inverse[composite[IMAGE[CAP], CART]], binclosed[CAP]]}] // Reverse

Out[20]= subclass[image[IMAGE[CAP], image[CART, cart[TOPS, TOPS]]], binclosed[CAP]] == True

In[21]:= subclass[image[IMAGE[CAP], image[CART, cart[TOPS, TOPS]]], binclosed[CAP]] := True
```

Theorem. Topologies generated by the combinatorial intersections of topologies.

```
In[25]:= SubstTest[implies, subclass[u, v], subclass[image[t, u], image[t, v]],
           {t → UCLOSURE, u -> image[IMAGE[CAP], image[CART, cart[TOPS, TOPS]]],
            v -> binclosed[CAP]}] // Reverse

Out[25]= subclass[image[UCLOSURE, image[IMAGE[CAP], image[CART, cart[TOPS, TOPS]]]], TOPS] == True

In[26]:= subclass[
           image[UCLOSURE, image[IMAGE[CAP], image[CART, cart[TOPS, TOPS]]]], TOPS] := True
```

Corollary. The Uclosure of the combinatorial intersection of two topologies is a topology.

```
In[28]:= SubstTest[implies, and[member[u, v], subclass[v, w]],
           member[u, w], {u → pair[top[x], top[y]], v → cart[TOPS, TOPS],
            w → image[inverse[composite[UCLOSURE, IMAGE[CAP], CART]], TOPS]}] // Reverse

Out[28]= member[Uclosure[image[CAP, cart[top[x], top[y]]]], TOPS] == True

In[29]:= member[Uclosure[image[CAP, cart[top[x_], top[y_]]]], TOPS] := True
```

Corollary. (Eliminating the **top** wrapper.)

```
In[30]:= SubstTest[implies, and[equal[x, top[u]], equal[y, top[v]]],
           member[Uclosure[image[CAP, cart[x, y]]], TOPS], {u → x, v → y}] // Reverse

Out[30]= or[member[Uclosure[image[CAP, cart[x, y]]], TOPS],
           not[member[x, TOPS]], not[member[y, TOPS]]] == True

In[31]:= or[member[Uclosure[image[CAP, cart[x_, y_]]], TOPS],
           not[member[x_, TOPS]], not[member[y_, TOPS]]] := True
```