sethood of image[RANK, x]

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2006 March 2

```
In[1]:= SetDirectory["l:"]; << goedel79.01a; << tools.m

:Package Title: goedel79.01a           2006 March 1 at 6:30 p.m.

It is now:  2006 Mar 2 at 21:11

Loading Simplification Rules

TOOLS.M                      Revised 2006 February 3

weightlimit = 40
```

summary

Although the **RANK** function is not one-to-one, its inverse is thin:

```
In[2]:= thin[inverse[RANK]]

Out[2]= True
```

This fact is exploited in this notebook to derive a sethood rule for **image[RANK, x]**. The class of the ranks of the regular members of a class **x** is a set if and only if the class of the regular members of **x** is a set.

derivation

Since the domain of **RANK** is the class **REGULAR**, one has:

```
In[3]:= SubstTest[implies, subclass[u, v], subclass[image[u, x], image[v, x]],
          {u -> id[REGULAR], v -> composite[inverse[RANK], RANK]}]

Out[3]= subclass[intersection[REGULAR, x], image[inverse[RANK], image[RANK, x]]] == True

In[4]:= (% /. x → x_) /. Equal → SetDelayed
```

Using the subset theorem yields:

```
In[5]:= SubstTest[implies, and[subclass[u, v], member[v, V]], member[u, V],
          {u -> intersection[REGULAR, x], v -> image[inverse[RANK], image[RANK, x]]}]

Out[5]= or[member[intersection[REGULAR, x], V],
          not[member[image[inverse[RANK], image[RANK, x]], V]]] == True
```

```
In[6]:= (% /. x → x_) /. Equal → SetDelayed
```

Combining the above results yields an implication in one direction:

```
In[7]:= Map[not, SubstTest[and, implies[p1, p2], implies[p2, p3], not[implies[p1, p3]],
          {p1 → member[image[RANK, x], V], p2 → member[image[inverse[RANK], image[RANK, x]], V],
           p3 → member[intersection[REGULAR, x], V]}]]

Out[7]= or[member[intersection[REGULAR, x], V], not[member[image[RANK, x], V]]] == True

In[8]:= (% /. x → x_) /. Equal → SetDelayed
```

Since imaging with the **RANK** function simply ignores the irregular members, an application of the axiom of replacement yields a reverse implication:

```
In[9]:= SubstTest[implies, member[y, V], member[image[RANK, y], V], y → intersection[REGULAR, x]]

Out[9]= or[member[image[RANK, x], V], not[member[intersection[REGULAR, x], V]]] == True

In[10]:= (% /. x → x_) /. Equal → SetDelayed
```

The sethood rule is obtained by combining the above implications:

```
In[11]:= equiv[member[image[RANK, x], V], member[intersection[REGULAR, x], V]]

Out[11]= True

In[12]:= member[image[RANK, x_], V] := member[intersection[REGULAR, x], V]
```