the integer one divides all integers

Johan G. F. Belinfante
2006 December 18

```
In[1]:= SetDirectory["l:"]; << goedel88.18a; << tools.m

        :Package Title: goedel88.18a       2006 December 18 at 9:00 a.m.

        It is now:  2006 Dec 18 at 9:34

        Loading Simplification Rules

        TOOLS.M                       Revised 2006 December 17

        weightlimit = 40
```

summary

In this notebook it is shown that every integer is divisible by the integer **+1** (**= plus[set[0]] = composite[id[omega], SUCC]**). To make sense of what is going on here, one must understand that the integer divisibility relation **INTDIV** has been defined as the union of all endomorphisms of **INTADD**. At this point, the theoy of multiplication for integers has not yet been developed, and indeed the result derived here is needed for that development.

derivation

By definition, the integer divisibility relation is:

```
In[2]:= U[binhom[INTADD, INTADD]]

Out[2]= INTDIV
```

Since the sum of two endomorphisms is another, the integer divisibility relation satisfies a corresponding property:

```
In[3]:= ImageComp[cross[inverse[DUP], INTADD], inverse[E],
          image[CROSS, cart[binhom[INTADD, INTADD], binhom[INTADD, INTADD]]]] // Reverse

Out[3]= composite[INTADD, intersection[
           composite[inverse[FIRST], INTDIV], composite[inverse[SECOND], INTDIV]]] == INTDIV

In[4]:= composite[INTADD, intersection[composite[inverse[FIRST], INTDIV],
           composite[inverse[SECOND], INTDIV]]] := INTDIV
```

Corollary. The set **image[INTDIV,set[x]]** of integers divisible by **x** is closed under addition.

```
In[5]:= ImageComp[composite[INTADD, cross[INTDIV, INTDIV]], DUP, set[x]] // Reverse

Out[5]= image[INTADD, cart[image[INTDIV, set[x]], image[INTDIV, set[x]]]] ==
         image[INTDIV, set[x]]

In[6]:= image[INTADD, cart[image[INTDIV, set[x_]], image[INTDIV, set[x_]]]] :=
         image[INTDIV, set[x]]
```

Lemma.

```
In[7]:= SubstTest[member, pair[x, x], composite[Id, w], w → INTDIV]

Out[7]= and[member[x, V], member[pair[x, x], INTDIV]] == member[pair[x, x], INTDIV]

In[8]:= and[member[x_, V], member[pair[x_, x_], INTDIV]] := member[pair[x, x], INTDIV]
```

Theorem. Every integer is divible by itself.

```
In[9]:= (member[x, fix[y]] // AssertTest // Reverse) /. y → INTDIV

Out[9]= member[pair[x, x], INTDIV] == member[x, Z]

In[10]:= member[pair[x_, x_], INTDIV] := member[x, Z]
```

A form of induction suitable for integers is applied to the class of integers divisible by one.

```
In[11]:= SubstTest[implies, and[member[composite[id[omega], SUCC], x],
            subclass[image[INTADD, cart[x, x]], x], subclass[image[INVERSE, x], x]],
          subclass[Z, x], x → image[INTDIV, set[plus[set[0]]]]] // Reverse

Out[11]= subclass[Z, image[INTDIV, set[composite[id[omega], SUCC]]]] == True

In[12]:= % /. Equal → SetDelayed
```

Theorem. Every integer is divisible by **+1**.

```
In[13]:= SubstTest[and, subclass[u, v], subclass[v, u],
          {u -> Z, v -> image[INTDIV, set[composite[id[omega], SUCC]]]}]

Out[13]= equal[Z, image[INTDIV, set[composite[id[omega], SUCC]]]] == True

In[14]:= image[INTDIV, set[composite[id[omega], SUCC]]] := Z
```