

# IRC commutes with IMAGE[IMAGE[id[x]]]

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```
In[1]:= SetDirectory["1:"]; << goedel.14feb01a
      :Package Title: goedel.14feb01a          2014 February 1 at 11:45 a.m.
      Loading takes about seventeen minutes, half that time due to builtin pauses.
      It is now: 2014 Feb 3 at 17:1
      Loading Simplification Rules
      TOOLS.M is now incorporated in the GOEDEL program as of 2010 September 3
      weightlimit = 40
      Loading completed.
      It is now: 2014 Feb 3 at 17:18
```

---

## summary

The content of Theorem 16 on page 51 of Kelley's book *General Topology* is that **IRC** commutes with **IMAGE[IMAGE[id[x]]]**. The function **IRC** takes any collection of sets to their relative complements in their union.

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## derivation

Theorem. (Temporary rewrite rule.)

```
In[9]:= composite[IMAGE[id[x]], RC[y]] // FastReifNormality
Out[9]= composite[IMAGE[id[x]], RC[y]] ==
      composite[id[image[V, set[y]]], RC[intersection[x, y]], IMAGE[id[x]], id[P[y]]]
In[10]:= composite[IMAGE[id[x_]], RC[y_]] :=
      composite[id[image[V, set[y]]], RC[intersection[x, y]], IMAGE[id[x]], id[P[y]]]
```

Corollary. (Temporary rewrite rule.)

```
In[13]:= ImageComp[IMAGE[id[x]], RC[U[setpart[y]]], setpart[y]] // Reverse
Out[13]= image[IMAGE[id[x]], image[RC[U[setpart[y]]], setpart[y]]] ==
      image[RC[intersection[x, U[setpart[y]]], image[IMAGE[id[x]], setpart[y]]]
```

```
In[14]:= image[IMAGE[id[x_]], image[RC[U[setpart[y_]]], setpart[y_]] :=
        image[RC[intersection[x, U[setpart[y]]], image[IMAGE[id[x]], setpart[y]]]
```

Observations.

```
In[16]:= APPLY[composite[IRC, IMAGE[IMAGE[id[x]]], setpart[y]]
```

```
Out[16]= image[RC[intersection[x, U[setpart[y]]], image[IMAGE[id[x]], setpart[y]]]
```

```
In[17]:= APPLY[composite[IMAGE[IMAGE[id[x]]], IRC, setpart[y]]
```

```
Out[17]= image[RC[intersection[x, U[setpart[y]]], image[IMAGE[id[x]], setpart[y]]]
```

Lemma.

```
In[20]:= dif[composite[IRC, IMAGE[IMAGE[id[x]]],
        composite[IMAGE[IMAGE[id[x]]], IRC]] // FastReifNormality
```

```
Out[20]= intersection[composite[IRC, IMAGE[IMAGE[id[x]]],
        composite[complement[IMAGE[IMAGE[id[x]]], IRC]] == 0
```

```
In[21]:= (% /. x -> x_) /. Equal -> SetDelayed
```

Lemma.

```
In[22]:= SubstTest[empty, dif[u, v],
        {u -> composite[IRC, IMAGE[IMAGE[id[x]]], v -> composite[IMAGE[IMAGE[id[x]]], IRC]}]
```

```
Out[22]= subclass[composite[IRC, IMAGE[IMAGE[id[x]]],
        composite[IMAGE[IMAGE[id[x]]], IRC]] == True
```

```
In[23]:= (% /. x -> x_) /. Equal -> SetDelayed
```

Theorem.

```
In[25]:= SubstTest[implies, and[subclass[u, v], FUNCTION[v]],
        equal[u, composite[v, id[domain[u]]], {u -> composite[IRC, IMAGE[IMAGE[id[x]]],
        v -> composite[IMAGE[IMAGE[id[x]]], IRC]}] // Reverse
```

```
Out[25]= equal[composite[IRC, IMAGE[IMAGE[id[x]]], composite[IMAGE[IMAGE[id[x]]], IRC]] == True
```

```
In[30]:= composite[IRC, IMAGE[IMAGE[id[x_]]] := composite[IMAGE[IMAGE[id[x]], IRC]
```

## serendipity

Theorem.

```
In[55]:= equal[V, image[RC[x], y]] // AssertTest
```

```
Out[55]= equal[V, image[RC[x], y]] == False
```

```
In[56]:= equal[V, image[RC[x_], y_]] := False
```

Observation. The function **IRC** takes any collection of sets to their relative complements in their union.

```
In[57]:= equal[APPLY[IRC, x], image[RC[U[x]], x]]
```

```
Out[57]= member[x, V]
```