Theorem ISB5HER1

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```
In[1]:= SetDirectory["l:"]; << goedel76.05a; << tools.m

        :Package Title: goedel76.05a            2005 December 5 at 4:00 p.m.

        It is now:  2005 Dec 6 at 13:55

        Loading Simplification Rules

        TOOLS.M                      Revised 2005 October 25

        weightlimit = 40
```

summary

Theorem **ISB5HER1** in the **ON-4** group was proved using McCune's program **Otter** on 2000 November 1. This theorem is rederived here in the **GOEDEL** program using essentially the same argument.

two lemmas

The following argument was proved by **Otter** on the fly. It was not explicitly stated as a separate lemma.

```
In[3]:= SubstTest[implies, and[subclass[y, z], member[z, w]],
         member[y, image[inverse[S], w]], w → image[inverse[S], x]]

Out[3]= or[member[y, image[inverse[S], x]],
          not[member[z, image[inverse[S], x]]], not[subclass[y, z]]] == True

In[4]:= or[member[y_, image[inverse[S], x_]],
          not[member[z_, image[inverse[S], x_]]], not[subclass[y_, z_]]] := True
```

Lemma.

```
In[8]:= SubstTest[and, full[y], subclass[y, OMEGA], y -> intersection[OMEGA, x]]

Out[8]= subclass[U[intersection[OMEGA, x]], x] ==
         or[member[intersection[OMEGA, x], OMEGA], subclass[OMEGA, x]]

In[9]:= subclass[U[intersection[OMEGA, x_]], x_] :=
         or[member[intersection[OMEGA, x], OMEGA], subclass[OMEGA, x]]
```

main argument

The clause numbers from **Otter**'s proof are retained here to facilitate comparison with that proof. The variables **n** and **r** appear as Skolem constants in the **Otter** proof.

```
In[5]:= Map[not, SubstTest[and, implies[and[p4323, p4324], p4375],
          implies[p4324, p4367], implies[and[p4367, p4375], p4425],
          implies[and[p3251, p4425], p4471], not[implies[and[p3251, p4323, p4324], p4471]],
          {p3251 → equal[image[inverse[S], x], x],
           p4323 → member[n, r], p4324 → member[r, intersection[OMEGA, x]],
           p4325 → not[member[n, intersection[OMEGA, x]]],
           p4367 → member[r, image[inverse[S], x]], p4375 → subclass[n, r],
           p4425 → member[n, image[inverse[S], x]], p4471 → member[n, x]}]]

Out[5]= or[member[n, x], not[equal[x, image[inverse[S], x]]],
          not[member[n, r]], not[member[r, OMEGA]], not[member[r, x]]] == True

In[6]:= or[member[n_, x_], not[equal[x_, image[inverse[S], x_]]],
          not[member[n_, r_]], not[member[r_, OMEGA]], not[member[r_, x]]] := True
```

Removing the variables **n** and **r** yields:

```
In[17]:= Map[equal[0, composite[Id, complement[#]]] &, SubstTest[class, pair[n, r],
            or[member[n, x], not[equal[x, y]], not[member[n, r]], not[member[r, w]],
             not[member[r, x]]], {w → OMEGA, y → image[inverse[S], x]}]] // Reverse

Out[17]= or[member[intersection[OMEGA, x], OMEGA],
           not[subclass[image[inverse[S], x], x]], subclass[OMEGA, x]] == True

In[18]:= (% /. x → x_) /. Equal → SetDelayed
```

This is Theorem **ISB5HER1**.

```
In[20]:= (or[member[intersection[w, x], w],
             not[equal[image[inverse[S], x], x]], subclass[w, x]] // AssertTest) /. w → OMEGA

Out[20]= or[member[intersection[OMEGA, x], OMEGA],
           not[equal[x, image[inverse[S], x]]], subclass[OMEGA, x]] == True

In[21]:= or[member[intersection[OMEGA, x_], OMEGA],
           not[equal[x_, image[inverse[S], x_]]], subclass[OMEGA, x_]] := True
```

In the special case that **x** is a set, the alternative **subclass[OMEGA, x]** cannot hold, and one can derive a variable-free formula.

```
In[25]:= Map[equal[V, #] &, SubstTest[class, x, or[member[intersection[w, x], w],
             not[equal[image[inverse[S], x], x]], subclass[w, x]], w → OMEGA]] // Reverse

Out[25]= subclass[image[IMAGE[id[OMEGA]], fix[IMAGE[inverse[S]]]], OMEGA] == True
```

```
In[26]:= subclass[image[IMAGE[id[OMEGA]], fix[IMAGE[inverse[S]]]], OMEGA] := True
```