

FUNCTION rules for GLB[x] and LUB[x]

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```
In[1]:= SetDirectory["1:"]; << goedel73.27b; << tools.m

:Package Title: goedel73.27b          2005 September 27 at 2:20 p.m.

It is now: 2005 Sep 27 at 16:39

Loading Simplification Rules

TOOLS.M          Revised 2005 September 27

weightlimit = 40
```

summary

It is shown that if x is an antisymmetric relation, then $\text{GLB}[x]$ and $\text{LUB}[x]$ are functions.

FUNCTION rule for LUB

Lemma.

```
In[2]:= SubstTest[subclass, intersection[u, v], v,
  {u -> composite[id[range[x]], UB[x]], v -> composite[id[range[x]],
    complement[composite[complement[inverse[x]], id[range[x]], UB[x]]]}]}
```

```
Out[2]= subclass[composite[id[range[x]], UB[x], inverse[LUB[x]]], x] == True
```

```
In[3]:= (% /. x -> x_) /. Equal -> SetDelayed
```

Theorem.

```
In[4]:= SubstTest[implies, and[subclass[u, v], subclass[v, w]], subclass[u, w],
  {u -> composite[LUB[x], inverse[LUB[x]]],
  v -> composite[id[range[x]], UB[x], inverse[LUB[x]]], w -> x}]
```

```
Out[4]= subclass[composite[LUB[x], inverse[LUB[x]]], x] == True
```

```
In[5]:= subclass[composite[LUB[x_], inverse[LUB[x_]]], x_] := True
```

Theorem.

```
In[6]:= SubstTest[implies, and[subclass[u, v], subclass[v, w]], subclass[u, w],
  {u -> composite[LUB[x], inverse[LUB[x]]], v -> intersection[x, inverse[x]], w -> Id}]
Out[6]= or[FUNCTION[LUB[x]], not[subclass[intersection[x, inverse[x]], Id]]] == True
In[7]:= or[FUNCTION[LUB[x_]], not[subclass[intersection[x_, inverse[x_]], Id]]] := True
```

It is convenient to add this also as a conditional rewrite rule.

```
In[8]:= FUNCTION[LUB[x_]] := True /; subclass[intersection[x, inverse[x]], Id]
```

FUNCTION rule for GLB

Analogous rules hold for **GLB**.

```
In[9]:= SubstTest[subclass, composite[LUB[y], inverse[LUB[y]]], y, y -> inverse[x]]
Out[9]= subclass[composite[GLB[x], inverse[GLB[x]]], x] == True
In[10]:= subclass[composite[GLB[x_], inverse[GLB[x_]]], x_] := True
In[11]:= SubstTest[implies, subclass[intersection[y, inverse[y]], Id],
  FUNCTION[LUB[y]], y -> inverse[x]]
Out[11]= or[FUNCTION[GLB[x]], not[subclass[intersection[x, inverse[x]], Id]]] == True
In[12]:= or[FUNCTION[GLB[x_]], not[subclass[intersection[x_, inverse[x_]], Id]]] := True
In[13]:= FUNCTION[GLB[x_]] := True /; subclass[intersection[x, inverse[x]], Id]
```

an example

```
In[14]:= FUNCTION[GLB[DIV]]
Out[14]= True
In[15]:= FUNCTION[LUB[DIV]]
Out[15]= True
```