pair groupoid

Johan G. F. Belinfante
2003 July 7

```
In[1]:= << goedel52.s46; << tools.m

        :Package Title: goedel52.s46      2003 July 6 at 1:05 p.m.

        It is now:  2003 Jul 8 at 11:22

        Loading Simplification Rules

        TOOLS.M                         Revised 2003 July 8

        weightlimit = 40
```

■ summary

The pair groupoid is shown to be an example of an associative relation. It is closely related to the rotation invariant function **RIF**, which is not itself associative.

■ reference

http : // mathworld.wolfram.com / Groupoid.html

■ definition of the pair groupoid

```
In[2]:= class[pair[pair[pair[u, v], pair[w, x]], pair[y, z]],
            and[equal[y, u], equal[v, w], equal[x, z]]]

Out[2]= composite[RIF, cross[SWAP, SWAP]]

In[3]:= associative[%] // AssertTest

Out[3]= associative[composite[RIF, cross[SWAP, SWAP]]] == True

In[4]:= associative[composite[RIF, cross[SWAP, SWAP]]] := True

In[5]:= FUNCTION[composite[RIF, cross[SWAP, SWAP]]]

Out[5]= True
```

The domain of this associative operation is not a cartesian product:

```
In[6]:= domain[composite[RIF, cross[SWAP, SWAP]]]

Out[6]= composite[inverse[FIRST], SECOND]
```

■ the associated transitive relations

There is are two transitive relations associated with any associative relation. In the present case there are:

```
In[7]:= implies[associative[x], TRANSITIVE[composite[x, inverse[FIRST]]]]

Out[7]= True

In[8]:= composite[x, inverse[FIRST]] /. x -> composite[RIF, cross[SWAP, SWAP]]

Out[8]= composite[inverse[FIRST], FIRST]

In[9]:= implies[associative[x], TRANSITIVE[composite[x, inverse[SECOND]]]]

Out[9]= True

In[10]:= composite[x, inverse[SECOND]] /. x -> composite[RIF, cross[SWAP, SWAP]]

Out[10]= composite[inverse[SECOND], SECOND]
```

The **GOEDEL** program already knows these facts:

```
In[11]:= Map[TRANSITIVE,
           {composite[inverse[FIRST], FIRST], composite[inverse[SECOND], SECOND]}]

Out[11]= {True, True}
```

■ RIF itself is not associative

One might wonder whether the function **RIF** itself is associative. It is not.

```
In[12]:= associative[RIF] // AssertTest

Out[12]= associative[RIF] == False
```

The **flip** of any associative relation is associative. For the pair groupoid, the **flip** is:

```
In[13]:= associative[composite[SWAP, RIF]] // AssertTest

Out[13]= associative[composite[SWAP, RIF]] == True

In[14]:= associative[composite[SWAP, RIF]] := True
```

Some other examples about which one might be curious. The rotation of the pair groupoid is not associative:

```
In[15]:= associative[rotate[composite[RIF, cross[SWAP, SWAP]]]] // AssertTest

Out[15]= associative[composite[SWAP, RIF, cross[Id, SWAP]]] == False
```

Here is another rotation−invariantfunction, the **flip** of **RIF**. It is not associative, either.

```
In[16]:= rotate[composite[SWAP, RIF, cross[SWAP, SWAP]]]

Out[16]= composite[SWAP, RIF, cross[SWAP, SWAP]]

In[17]:= associative[flip[RIF]] // AssertTest

Out[17]= associative[composite[SWAP, RIF, cross[SWAP, SWAP]]] == False
```