quasigroup RIGHT and LEFT rules

Johan G. F. Belinfante
2008 November 30

```
In[1]:= SetDirectory["l:"]; << goedel.08nov29a;<< tools.m

        :Package Title: goedel.08nov29a          2008 November 29 at 10:55 p.m.

        It is now:  2008 Nov 30 at 21:25

        Loading Simplification Rules

        TOOLS.M                                    Revised 2008 October 21

        weightlimit = 40
```

summary

The **LEFT** and **RIGHT** multiplications for quasigroups and their rotations yield six functions which come in three pairs.

observations

Observations.

```
In[2]:= composite[into[quasigp[x]], LEFT[y]]

Out[2]= composite[inverse[LEFT[y]], inverse[quasigp[x]]]

In[3]:= composite[into[quasigp[x]], RIGHT[y]]

Out[3]= image[inverse[quasigp[x]], set[y]]

In[4]:= composite[over[quasigp[x]], LEFT[y]]

Out[4]= inverse[image[inverse[quasigp[x]], set[y]]]

In[5]:= composite[over[quasigp[x]], RIGHT[y]]

Out[5]= composite[inverse[RIGHT[y]], inverse[quasigp[x]]]
```

derivation of two new rules

Theorem.

```
In[6]:= SubstTest[composite, funpart[t], inverse[funpart[t]],
          t -> composite[into[quasigp[x]], LEFT[y]]] // Reverse

Out[6]= composite[inverse[LEFT[y]], inverse[quasigp[x]], quasigp[x], LEFT[y]] ==
         id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]

In[7]:= composite[inverse[LEFT[y_]], inverse[quasigp[x_]], quasigp[x_], LEFT[y_]] :=
         id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

Theorem.

```
In[8]:= SubstTest[composite, funpart[t], inverse[funpart[t]],
          t -> composite[over[quasigp[x]], RIGHT[y]]] // Reverse

Out[8]= composite[inverse[RIGHT[y]], inverse[quasigp[x]], quasigp[x], RIGHT[y]] ==
         id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]

In[9]:= composite[inverse[RIGHT[y_]], inverse[quasigp[x_]], quasigp[x_], RIGHT[y_]] :=
         id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

Comment: There are a total of six such equations, but the other ones are already available.

the six basic equations

Each of the six basic equations of quasigroup theory is a statement to the effect that one of the six **LEFT** and **RIGHT** multiplications is a function. These are:

(IL)

```
In[10]:= FUNCTION[composite[into[quasigp[x]], LEFT[y]]]

Out[10]= True

In[11]:= composite[composite[into[quasigp[x]], LEFT[y]], composite[quasigp[x], LEFT[y]]]

Out[11]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

(IR)

```
In[12]:= FUNCTION[composite[quasigp[x], RIGHT[y]]]

Out[12]= True

In[13]:= composite[composite[quasigp[x], RIGHT[y]], composite[over[quasigp[x]], RIGHT[y]]]

Out[13]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

(SL)

```
In[14]:= FUNCTION[composite[quasigp[x], LEFT[y]]]

Out[14]= True

In[15]:= composite[composite[quasigp[x], LEFT[y]], composite[into[quasigp[x]], LEFT[y]]]

Out[15]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

(SR)

```
In[16]:= FUNCTION[composite[quasigp[x], RIGHT[y]]]

Out[16]= True

In[17]:= composite[composite[quasigp[x], RIGHT[y]], composite[over[quasigp[x]], RIGHT[y]]]

Out[17]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

(DL)

```
In[18]:= FUNCTION[composite[over[quasigp[x]], LEFT[y]]]

Out[18]= True

In[19]:= composite[composite[over[quasigp[x]], LEFT[y]], composite[into[quasigp[x]], RIGHT[y]]]

Out[19]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

(DR)

```
In[20]:= FUNCTION[composite[into[quasigp[x]], RIGHT[y]]]

Out[20]= True

In[21]:= composite[composite[into[quasigp[x]], RIGHT[y]], composite[over[quasigp[x]], LEFT[y]]]

Out[21]= id[intersection[image[V, intersection[range[quasigp[x]], set[y]]], range[quasigp[x]]]]
```

Comment. Each of these three identities reduces to **id[range[quasigp[x]]]** if **y** belongs to **range[quasigroup[x]]**, and is **0** otherwise.