subvariance under inverse[PS]

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```
In[1]:= SetDirectory["l:"]; << goedel79.01a; << tools.m

        :Package Title: goedel79.01a           2006 March 1 at 6:30 p.m.

        It is now:  2006 Mar 2 at 21:33

        Loading Simplification Rules

        TOOLS.M                      Revised 2006 February 3

        weightlimit = 40
```

summary

The class of all finite subsets of **x** is subvariant under **inverse[PS]** if and only if **x** is not a finite set.

two general lemmas

The following two lemmas are derived in a similar fashion.

```
In[2]:= SubstTest[member, U[y], V, y → intersection[FINITE, P[x]]] // Reverse

Out[2]= member[intersection[FINITE, P[x]], V] == member[x, V]

In[3]:= member[intersection[FINITE, P[x_]], V] := member[x, V]

In[4]:= SubstTest[member, U[y], FINITE, y → intersection[FINITE, P[x]]] // Reverse

Out[4]= member[intersection[FINITE, P[x]], FINITE] == member[x, FINITE]

In[5]:= member[intersection[FINITE, P[x_]], FINITE] := member[x, FINITE]
```

derivation

If **y** is a finite subset of a class **x**, and if there is an element **w** in **dif[x, y]**, then adjoining this element produces a bigger finite subset of **x**.

```
In[6]:= SubstTest[implies, member[pair[u, v], composite[s, id[t]]], member[v, image[s, t]],
          {s → inverse[PS], u → union[y, set[w]], v → y, t → intersection[FINITE, P[x]]}]

Out[6]= or[member[w, y], member[y, image[inverse[PS], intersection[FINITE, P[x]]]],
          not[member[w, x]], not[member[y, FINITE]], not[subclass[y, x]]] == True

In[7]:= (% /. {w → w_, x → x_, y → y_}) /. Equal → SetDelayed
```

Technical lemma.

```
In[8]:= not[member[y, image[inverse[PS], intersection[FINITE, P[x]]]]] // AssertTest // Reverse

Out[8]= subclass[intersection[FINITE, image[S, set[y]], P[x]], set[y]] ==
         not[member[y, image[inverse[PS], intersection[FINITE, P[x]]]]]

In[9]:= subclass[intersection[FINITE, image[S, set[y_]], P[x_]], set[y_]] :=
         not[member[y, image[inverse[PS], intersection[FINITE, P[x]]]]]
```

Eliminating the variable **w** yields:

```
In[10]:= Map[equal[V, #] &, SubstTest[class, w,
            or[member[w, y], member[y, u], not[member[w, x]], not[member[y, v]]],
            {u -> image[inverse[PS], intersection[FINITE, P[x]]],
             v → intersection[FINITE, P[x]]}]] // Reverse

Out[10]= or[equal[x, y], member[y, image[inverse[PS], intersection[FINITE, P[x]]]],
           not[member[y, FINITE]], not[subclass[y, x]], subclass[x, y]] == True

In[11]:= (% /. {x → x_, y → y_}) /. Equal → SetDelayed
```

If **x** is infinite, and **y** is finite, then **x** and **y** cannot be equal, and **x** is not a subclass of **y**.

```
In[12]:= Map[not, SubstTest[and, implies[and[p1, p2], p3],
           implies[and[p1, p2], p4], implies[and[p1, p3, p4], p5],
           not[implies[and[p1, p2], p5]], {p1 → member[y, intersection[FINITE, P[x]]],
            p2 → not[member[x, FINITE]], p3 → not[equal[x, y]], p4 → not[subclass[x, y]],
            p5 → member[y, image[inverse[PS], intersection[FINITE, P[x]]]]}]]

Out[12]= or[member[x, FINITE], member[y, image[inverse[PS], intersection[FINITE, P[x]]]],
           not[member[y, FINITE]], not[subclass[y, x]]] == True

In[13]:= (% /. {x → x_, y → y_}) /. Equal → SetDelayed
```

Theorem. If **x** is an infinite set, then the collection of finite subsets of **x** is subvariant under **inverse[PS]**.

```
In[14]:= Map[equal[V, #] &,
           SubstTest[class, y, or[member[x, FINITE], member[y, u], not[member[y, v]]],
            {u -> image[inverse[PS], intersection[FINITE, P[x]]],
             v -> intersection[FINITE, P[x]]}]] // Reverse

Out[14]= or[member[x, FINITE], subclass[intersection[FINITE, P[x]],
            image[inverse[PS], intersection[FINITE, P[x]]]]] == True

In[15]:= (% /. x → x_) /. Equal → SetDelayed
```

A better result is obtained by combining this result with the converse:

```
In[16]:= SubstTest[implies, and[member[u, v], subclass[v, w]],
          member[u, w], {u → intersection[FINITE, P[x]],
           v → subvar[inverse[PS]], w → union[set[0], complement[FINITE]]}]

Out[16]= or[not[member[x, FINITE]], not[subclass[intersection[FINITE, P[x]],
             image[inverse[PS], intersection[FINITE, P[x]]]]]] == True

In[17]:= (% /. x → x_) /. Equal → SetDelayed

In[18]:= equiv[subclass[intersection[FINITE, P[x]],
           image[inverse[PS], intersection[FINITE, P[x]]]], not[member[x, FINITE]]]

Out[18]= True
```

This justifies the following rewrite rule, which says that the class of finite subsets of a class **x** is subvariant under **inverse[PS]** if and only if **x** is not a finite set.

```
In[19]:= subclass[intersection[FINITE, P[x_]],
           image[inverse[PS], intersection[FINITE, P[x_]]]] := not[member[x, FINITE]]
```

Corollary. (Special case of this rule.)

```
In[20]:= SubstTest[subclass, intersection[FINITE, P[x]],
          image[inverse[PS], intersection[FINITE, P[x]]], x → V]

Out[20]= subclass[FINITE, image[inverse[PS], FINITE]] == True

In[21]:= subclass[FINITE, image[inverse[PS], FINITE]] := True
```