

## two expressions of the form $f[w] \subset f[x] \cup f[y]$

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```
In[1]:= SetDirectory["1:"]; << goedel.10jan01a; << tools.m

:Package Title: goedel.10jan01a          2010 January 1 at 5:45 p.m.

It is now: 2010 Jan 3 at 8:20

Loading Simplification Rules

TOOLS.M                                Revised 2009 December 17

weightlimit = 40
```

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### summary

Two curious rewrite rules are derived for expressions of the form  $f[w] \subset f[x] \cup f[y]$ . One is for the case that  $f$  is the power class constructor, and the other is for the case that  $f$  is the cartesian square constructor. The one rule implies the other.

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### derivation

Theorem. The case of cartesian squares.

```
In[2]:= SubstTest[subclass, image[E, u], image[E, v],
  {u -> cartsq[w], v -> union[cartsq[x], cartsq[y]]}]

Out[2]= subclass[cart[w, w], union[cart[x, x], cart[y, y]]] = or[subclass[w, x], subclass[w, y]]

In[3]:= subclass[cart[w_, w_], union[cart[x_, x_], cart[y_, y_]]] :=
  or[subclass[w, x], subclass[w, y]]
```

Corollary. The case of power classes.

```
In[4]:= SubstTest[subclass, P[w], cliques[u], u -> union[cartsq[x], cartsq[y]]] // Reverse

Out[4]= subclass[P[w], union[P[x], P[y]]] = or[subclass[w, x], subclass[w, y]]

In[5]:= subclass[P[w_], union[P[x_], P[y_]]] := or[subclass[w, x], subclass[w, y]]
```