T2 and transvar

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2014 February 12

```
In[1]:= SetDirectory["l:"]; << goedel.14feb09a

        :Package Title: goedel.14feb09a                2014 February 9 at 5:50 p.m.

        Loading takes about seventeen minutes, half that time due to builtin pauses.

        It is now:  2014 Feb 12 at 12:6

        Loading Simplification Rules

        TOOLS.M is now incorporated in the GOEDEL program as of 2010 September 3

        weightlimit = 40

        Loading completed.

        It is now:  2014 Feb 12 at 12:24
```

summary

The **T2** separation condition in topology is technically complicated because this condition involves four occurrences of the variable for the topology.

```
In[2]:= equiv[member[t, T2], and[member[t, V], subclass[composite[id[U[t]], Di, id[U[t]]],
            composite[inverse[E], id[t], DISJOINT, id[t], E]]]] // not // not

Out[2]= True
```

One way to simplify this condition is to introduce the cartesian square of **t** as a new variable **x = t × t**. The condition on **t** can then be rewritten as a condition on **x** that involves only two occurrences of the variable **x**. This observation is used to derive a formula for **T2** involving a **transvar[x, y]** expression. A consequence is that the class **T2** is closed under unions of chains.

derivation

Lemma.

```
In[3]:= member[t, symdif[T2,
          fix[image[inverse[CART], transvar[composite[inverse[E], IMAGE[id[Di]], CART],
            composite[inverse[E], CART, id[DISJOINT]]]]]]] // NotNotTest

Out[3]= or[and[member[t, T2], not[subclass[composite[id[U[t]], Di, id[U[t]]],
            composite[inverse[E], id[t], DISJOINT, id[t], E]]]],
         and[member[t, V], not[member[t, T2]], subclass[composite[id[U[t]], Di, id[U[t]]],
           composite[inverse[E], id[t], DISJOINT, id[t], E]]]] == False

In[4]:= (% /. t → t_) /. Equal → SetDelayed
```

Theorem. A formula for **T2**.

```
In[5]:= Map[equal[V, #] &, SubstTest[class, t, not[member[t, w]],
         w -> symdif[T2, fix[image[inverse[CART], transvar[composite[inverse[E],
               IMAGE[id[Di]], CART], composite[inverse[E], CART, id[DISJOINT]]]]]]]]

Out[5]= equal[T2, fix[image[inverse[CART], transvar[composite[inverse[E], IMAGE[id[Di]], CART],
            composite[inverse[E], CART, id[DISJOINT]]]]]] == True

In[6]:= fix[image[inverse[CART], transvar[composite[inverse[E], IMAGE[id[Di]], CART],
           composite[inverse[E], CART, id[DISJOINT]]]]] := T2
```

Lemma.

```
In[10]:= SubstTest[subclass, Uchains[fix[image[inverse[CART], t]]],
           fix[image[inverse[CART], Uchains[t]]], t → Uclosure[x]] // Reverse

Out[10]= subclass[Uchains[fix[image[inverse[CART], Uclosure[x]]]],
           fix[image[inverse[CART], Uclosure[x]]]] == True

In[11]:= (% /. x → x_) /. Equal → SetDelayed
```

Theorem.

```
In[14]:= equal[Uchains[fix[image[inverse[CART], Uclosure[x]]]],
           fix[image[inverse[CART], Uclosure[x]]]] // AssertTest

Out[14]= equal[fix[image[inverse[CART], Uclosure[x]]],
           Uchains[fix[image[inverse[CART], Uclosure[x]]]]] == True

In[16]:= Uchains[fix[image[inverse[CART], Uclosure[x_]]]] :=
          fix[image[inverse[CART], Uclosure[x]]]
```

Corollary.

```
In[18]:= SubstTest[Uchains,
           fix[image[inverse[CART], Uclosure[t]]], t → transvar[x, y]] // Reverse

Out[18]= Uchains[fix[image[inverse[CART], transvar[x, y]]]] ==
          fix[image[inverse[CART], transvar[x, y]]]

In[19]:= Uchains[fix[image[inverse[CART], transvar[x_, y_]]]] :=
          fix[image[inverse[CART], transvar[x, y]]]
```

Theorem. The class **T2** is closed under chain-unions.

```
In[21]:= SubstTest[Uchains, fix[image[inverse[CART], transvar[x, y]]],
           {x -> composite[inverse[E], IMAGE[id[Di]], CART],
            y -> composite[inverse[E], CART, id[DISJOINT]]}] // Reverse

Out[21]= Uchains[T2] == T2

In[22]:= Uchains[T2] := T2
```

comment

Observation. The following holds, but no interesting consequences of it were obtained.

```
In[30]:= intersection[image[CART, Id], transvar[composite[inverse[E], IMAGE[id[Di]], CART],
           composite[inverse[E], CART, id[DISJOINT]]]]

Out[30]= image[CART, id[T2]]
```

Observation. One can use this to deduce the following fact, but this result is already known.

```
In[31]:= SubstTest[member, cart[x, x], intersection[u, v],
          {u -> image[CART, Id], v -> transvar[composite[inverse[E], IMAGE[id[Di]], CART],
             composite[inverse[E], CART, id[DISJOINT]]]}]

Out[31]= and[member[x, V], subclass[composite[id[U[x]], Di, id[U[x]]],
            composite[inverse[E], id[x], DISJOINT, id[x], E]]] == member[x, T2]
```

Theorem. An amusing result.

```
In[34]:= SubstTest[implies, and[equal[Uchains[x], x], equal[Uchains[y], y]],
           equal[Uchains[intersection[x, y]], intersection[x, y]],
           {x -> image[CART, Id], y -> transvar[composite[inverse[E], IMAGE[id[Di]], CART],
              composite[inverse[E], CART, id[DISJOINT]]]}] // Reverse

Out[34]= equal[image[CART, id[T2]], Uchains[image[CART, id[T2]]]] == True

In[36]:= Uchains[image[CART, id[T2]]] := image[CART, id[T2]]
```