

# ranges of subgroups are closed under chain unions

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```
In[1]:= SetDirectory["1:"]; << goedel.11nov17a
      :Package Title: goedel.11nov17a          2011 November 17 at 4:00 p.m.
      Loading takes about thirteen minutes, half that time due to builtin pauses.
      It is now: 2011 Nov 19 at 19:39
      Loading Simplification Rules
      TOOLS.M is now incorporated in the GOEDEL program as of 2010 September 3
      weightlimit = 40
      Loading completed.
      It is now: 2011 Nov 19 at 19:52
```

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## summary

The union of a nonempty chain of ranges of subgroups of a group is the range of a subgroup.

---

## derivation

Theorem.

```
In[9]:= SubstTest[union, intersection[binclosed[u], complement[set[0]], fix[IMAGE[v]]],
      set[0], {u → gp[x], v → inv[gp[x]]}]
```

```
Out[9]= intersection[binclosed[gp[x]], fix[IMAGE[inv[gp[x]]]]] ==
      union[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]]], set[0]]
```

```
In[10]:= intersection[binclosed[gp[x_]], fix[IMAGE[inv[gp[x_]]]]] :=
      union[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]]], set[0]]
```

Theorem.

```
In[13]:= SubstTest[Uchains, intersection[binclosed[u], fix[IMAGE[v]]],
      {u → gp[x], v → inv[gp[x]]}] // Reverse
```

```
Out[13]= Uchains[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]]]] ==
      union[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]]], set[0]]
```

```
In[14]:= Uchains[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x_]]]] :=
        union[image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]]], set[0]]
```

Corollary.

```
In[17]:= SubstTest[Uchains,
        image[IMAGE[SECOND], intersection[GROUPS, P[gp[x]]], x → INTADD] // Reverse
```

```
Out[17]= Uchains[image[VERTSECT[INTDIV], Z]] == range[VERTSECT[INTDIV]]
```

```
In[18]:= Uchains[image[VERTSECT[INTDIV], Z]] := range[VERTSECT[INTDIV]]
```

Corollary.

```
In[20]:= SubstTest[Uchains, union[x, set[0]], x → image[VERTSECT[INTDIV], Z]] // Reverse
```

```
Out[20]= Uchains[range[VERTSECT[INTDIV]]] == range[VERTSECT[INTDIV]]
```

```
In[21]:= Uchains[range[VERTSECT[INTDIV]]] := range[VERTSECT[INTDIV]]
```

## similar results for DIV

Theorem.

```
In[26]:= SubstTest[Uchains, intersection[binclosed[u], binclosed[v], P[w]],
        {u → NATADD, v → rotate[NATADD], w → omega}] // Reverse
```

```
Out[26]= Uchains[range[VERTSECT[DIV]]] == range[VERTSECT[DIV]]
```

```
In[27]:= Uchains[range[VERTSECT[DIV]]] := range[VERTSECT[DIV]]
```

Corollary.

```
In[29]:= SubstTest[Uchains, dif[x, set[0]], x → range[VERTSECT[DIV]]] // Reverse
```

```
Out[29]= Uchains[image[VERTSECT[DIV], omega]] == range[VERTSECT[DIV]]
```

```
In[30]:= Uchains[image[VERTSECT[DIV], omega]] := range[VERTSECT[DIV]]
```