3, 5 and 7 are primes

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```
In[1]:= SetDirectory["i:"]; << goedel71.24a; << tools.m

        :Package Title: goedel71.24a          2005 July 24 at 6:35 p.m.

        It is now:  2005 Jul 25 at 9:42

        Loading Simplification Rules

        TOOLS.M                       Revised 2005 July 18

        weightlimit = 40
```

---

summary

This notebook contains brute force derivations of the facts that **3**, **5** and **7** are primes, but **4**, **6**, **8**, **9** and **10** are not. The execution times for the non-primes is less than that for the primes, but both grow rapidly.

---

general results

The divisors of a number are no greater than that number. The following formulation of this observation will be used to show that **3** and **5** are primes.

```
In[2]:= equiv[subclass[image[inverse[DIV], set[x]], succ[x]],
           or[not[member[x, omega]], not[equal[0, x]]]] // not // not

Out[2]= True

In[3]:= subclass[image[inverse[DIV], set[x_]], succ[x_]] :=
         or[not[equal[0, x]], not[member[x, omega]]]
```

The only divisors of a prime are **1** and itself. This observation will be used to show that **4** is not a prime.

```
In[4]:= SubstTest[implies, and[member[u, v], equal[v, w]], member[u, w],
          {u → x, v → image[inverse[DIV], set[y]], w → set[set[0], y]}] // MapNotNot

Out[4]= or[equal[x, y], equal[x, set[0]],
          not[member[y, PRIMES]], not[member[pair[x, y], DIV]]] == True

In[5]:= or[equal[x_, y_], equal[x_, set[0]],
          not[member[y_, PRIMES]], not[member[pair[x_, y_], DIV]]] := True
```

3, 5 and 7 are primes

Showing that **3** is prime takes only 0.2 seconds.

```
In[6]:= Map[subclass[image[inverse[DIV], set[succ[succ[set[0]]]]],
            dif[succ[succ[succ[set[0]]]], #]] &,
          intersection[complement[set[set[0], succ[succ[set[0]]]]],
            succ[succ[succ[set[0]]]]] // Normality] // Timing

Out[6]= {0.203 Second, member[succ[succ[set[0]]], PRIMES] == True}

In[7]:= member[succ[succ[set[0]]], PRIMES] := True
```

Showing that **5** is prime takes nine times longer.

```
In[8]:= Map[subclass[image[inverse[DIV], set[succ[succ[succ[succ[set[0]]]]]],
            dif[succ[succ[succ[succ[succ[set[0]]]]], #]] &,
          intersection[complement[set[set[0], succ[succ[succ[succ[set[0]]]]]],
            succ[succ[succ[succ[succ[set[0]]]]]] // Normality] // Timing

Out[8]= {1.656 Second, member[succ[succ[succ[succ[set[0]]]], PRIMES] == True}

In[9]:= member[succ[succ[succ[succ[set[0]]]], PRIMES] := True
```

The execution time for the case of **7** is up by another factor of nine.

```
In[10]:= Map[subclass[image[inverse[DIV],
             set[succ[succ[succ[succ[succ[succ[set[0]]]]]]]], dif[
             succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]], #]] &, intersection[
            complement[set[set[0], succ[succ[succ[succ[succ[succ[set[0]]]]]]]],
            succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]] // Normality] // Timing

Out[10]= {15.016 Second,
          member[succ[succ[succ[succ[succ[succ[set[0]]]]]]], PRIMES] == True}

In[11]:= member[succ[succ[succ[succ[succ[succ[set[0]]]]]]], PRIMES] := True
```

4, 6, 8, 9 and 10 are not primes

Since **2** divides **4**, it follows that **4** is not a prime.

```
In[12]:= Map[not, SubstTest[implies, and[member[pair[x, y], DIV], member[y, PRIMES]],
           or[equal[x, y], equal[x, set[0]]],
           {x → succ[set[0]], y → succ[succ[succ[set[0]]]]}]] // Timing
```

```
Out[12]= {0.015 Second, member[succ[succ[succ[set[0]]]], PRIMES] == False}
```

```
In[13]:= member[succ[succ[succ[set[0]]]], PRIMES] := False
```

Since **2** divides **6**, it follows that **6** is not a prime.

```
In[14]:= Map[not, SubstTest[implies, and[member[pair[x, y], DIV], member[y, PRIMES]],
           or[equal[x, y], equal[x, set[0]]],
           {x → succ[set[0]], y → succ[succ[succ[succ[succ[set[0]]]]]]}]] // Timing
```

```
Out[14]= {0.172 Second, member[succ[succ[succ[succ[succ[set[0]]]]], PRIMES] == False}
```

```
In[15]:= member[succ[succ[succ[succ[succ[set[0]]]]], PRIMES] := False
```

Since **2** divides **8**, it follows that **8** is not a prime.

```
In[16]:= Map[not, SubstTest[implies, and[member[pair[x, y], DIV], member[y, PRIMES]],
           or[equal[x, y], equal[x, set[0]]], {x → succ[set[0]],
            y → succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]]}]] // Timing
```

```
Out[16]= {1.032 Second,
          member[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]], PRIMES] == False}
```

```
In[17]:= member[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]], PRIMES] := False
```

Since **3** divides **9**, it follows that **9** is not a prime.

```
In[18]:= Map[not, SubstTest[implies, and[member[pair[x, y], DIV], member[y, PRIMES]],
           or[equal[x, y], equal[x, set[0]]], {x → succ[succ[set[0]]],
            y → succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]]]}]] // Timing
```

```
Out[18]= {1.031 Second, member[
           succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]], PRIMES] == False}
```

```
In[19]:= member[succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]], PRIMES] :=
          False
```

Since **2** divides **10**, it follows that **10** is not a prime. For this case, the execution time is seven seconds.

```
In[20]:= Map[not, SubstTest[implies, and[member[pair[x, y], DIV], member[y, PRIMES]],
           or[equal[x, y], equal[x, set[0]]], {x → succ[set[0]], y → succ[
             succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]]]}]] // Timing

Out[20]= {7.094 Second,
          member[succ[succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]]]],
            PRIMES] == False}

In[21]:= member[succ[succ[succ[succ[succ[succ[succ[succ[succ[set[0]]]]]]]]]],
           PRIMES] := False
```