nine TWIST formula

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```
<< goedel52.q85; << tools.m

:Package Title: goedel52.q85          2002 December 28 at 11:58 p.m.

It is now:  2002 Dec 30 at 10:31

Loading Simplification Rules

TOOLS.M                    Revised 2002 December 27

weightlimit = 40
```

■ summary

This notebook contains an exercise in which various forms of **Normality** are used to derive an intriguing formula in which nine copies of **TWIST** conspire to cancel out. The formula itself can be understood from the following observation:

```
image[composite[cross[TWIST, TWIST], TWIST],
    cart[cart[cart[1, 2], cart[3, 4]], cart[cart[5, 6], cart[7, 8]]]]

cart[cart[cart[1, 5], cart[2, 6]], cart[cart[3, 7], cart[4, 8]]]
```

The function **composite[cross[TWIST,TWIST],TWIST]** is a representation of the permutation **(253)(467)**, which has period three. So it is reasonable to expect that three applications of this function will be an identity function, and this is indeed the case, as will be verified below. The derivation is fairly time−consuming and requiries two lemmas that seem interesting in their own right.

■ lemma 1: formula involving TWIST and cross

The following formula could be composed with **TWIST** on either side, but it would then be unclear how to orient the resulting formula. The orientation issue is sidestepped here by having the formula eliminate **TWIST**.

```
composite[TWIST, cross[cross[u, v], cross[x, y]], TWIST] // VSTerNormality // Timing

{53.75 Second, composite[TWIST, cross[cross[u, v], cross[x, y]], TWIST] ==
  cross[cross[u, x], cross[v, y]]}

composite[TWIST, cross[cross[u_, v_], cross[x_, y_]], TWIST] :=
 cross[cross[u, x], cross[v, y]]
```

■ lemma 2: twist singleton formula

In this section a formula for **twist[singleton[x]]** is derived. There is an analogous formula for **rotate[singleton[x]]**, but it caused delays for examples similar to the following one:

```
class[pair[x, y], member[y, twist[singleton[x]]]] // Timing
```

{40.61 Second, TWIST}

The **twist** formula derived below does not suffer from this problem. To get the best result, we note that the following is recognized to be true:

```
equal[cart[cart[singleton[first[first[x]]], singleton[first[second[x]]]],
  cart[singleton[second[first[x]]], intersection[
    image[V, singleton[first[first[x]]]], singleton[second[second[x]]]]]],
    cart[cart[singleton[first[first[x]]], singleton[first[second[x]]]],
  cart[singleton[second[first[x]]], singleton[second[second[x]]]]]]
```

True

This justifies adding the corresponding rewrite rule:

```
cart[cart[singleton[first[first[x_]]], singleton[first[second[x_]]]],
  cart[singleton[second[first[x_]]], intersection[
    image[V, singleton[first[first[x_]]]], singleton[second[second[x_]]]]]] :=
 cart[cart[singleton[first[first[x]]], singleton[first[second[x]]]],
   cart[singleton[second[first[x]]], singleton[second[second[x]]]]]
```

The desired formula is now derived:

```
twist[singleton[x]] // VSTriNormality // Timing
```

{10.469 Second, twist[singleton[x]] ==
  cart[cart[singleton[first[first[x]]], singleton[first[second[x]]]],
   cart[singleton[second[first[x]]], singleton[second[second[x]]]]]}

```
twist[singleton[x_]] :=
 cart[cart[singleton[first[first[x]]], singleton[first[second[x]]]],
   cart[singleton[second[first[x]]], singleton[second[second[x]]]]]
```

The example that caused some concern is repeated, and one finds that the execution time actually went down.

```
class[pair[x, y], member[y, twist[singleton[x]]]] // Timing
```

{25.375 Second, TWIST}

Comment: This example does not even yield the result **TWIST**   when the **simplify** flag is off.

■ nine TWIST formula

The main result takes a long time:

```
composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST],
   TWIST, cross[TWIST, TWIST], TWIST] // VSNormality // Timing
```

```
{161.75 Second,
 composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST],
   TWIST] == id[cart[cart[cart[V, V], cart[V, V]], cart[cart[V, V], cart[V, V]]]]}
```

The main formula can be derived in half the time if one turns **simplify** off.

```
simplify = False;
```

```
composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST],
   TWIST, cross[TWIST, TWIST], TWIST] // VSNormality // Timing
```

```
{73.734 Second,
 composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST],
   TWIST] == id[cart[cart[cart[V, V], cart[V, V]], cart[cart[V, V], cart[V, V]]]]}
```

For the following corollary, the **simplify** flag needs to be turned back on.

```
simplify = True;
```

```
Map[composite[TWIST, cross[TWIST, TWIST], #, TWIST] &, Last[%%]]
```

```
composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST]] ==
 composite[TWIST, cross[TWIST, TWIST], TWIST]
```

```
composite[cross[TWIST, TWIST], TWIST, cross[TWIST, TWIST]] :=
 composite[TWIST, cross[TWIST, TWIST], TWIST]
```