Corollary ON−1−B

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```
<< goedel52.r04; << tools.m

:Package Title: goedel52.r04          2003 January 19 at 9:25 p.m.

It is now:  2003 Jan 20 at 16:10

Loading Simplification Rules

TOOLS.M                       Revised 2002 December 27

weightlimit = 40
```

■ summary

This notebook contains an adaptation of **Otter**'s proof of Corollary **ON−1−B**. This simple corollary follows directly from the definition of the class **OMEGA** of ordinal numbers. In the **GOEDEL** program this membership rule is wrapped to prevent it from being routinely expanded, which would be a nuisance, but one can still view it easily by using **assert**.

```
member[x, OMEGA] // AssertTest

member[x, OMEGA] == and[member[x, V], subclass[intersection[FULL, P[x]], succ[x]]]
```

■ derivation

To use the definition of ordinal numbers, it is advantageous to introduce a variable **x** as follows:

```
SubstTest[implies, and[member[u, v], subclass[v, w]], member[u, w],
    {u -> 0, v -> intersection[FULL, P[x]], w -> succ[x]}]

or[equal[0, x], member[0, x], not[subclass[intersection[FULL, P[x]], succ[x]]]] == True

or[equal[0, x_], member[0, x_],
  not[subclass[intersection[FULL, P[x_]], succ[x_]]]] := True
```

With this temporary rule in place, one can deduce Corollary **ON−1−B** as follows:

```
Map[not, SubstTest[and, implies[p1, p2], implies[p2, p3], not[implies[p1, p3]],
        {p1 -> member[x, OMEGA], p2 -> subclass[intersection[FULL, P[x]], succ[x]],
            p3 -> member[0, succ[x]]}]]

or[equal[0, x], member[0, x], not[member[x, OMEGA]]] == True

or[equal[0, x_], member[0, x_], not[member[x_, OMEGA]]] := True
```