analysis of Otter's proof of Theorem ON−A−1

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```
<< goedel52.o14; << tools.m
```

```
:Package Title: goedel52.o14                        2002 May 29 at 6:15 a.m.

It is now:  2002 May 31 at 5:40

Loading Simplification Rules

TOOLS.M                   Revised 2002 May 22

weightlimit = 40
```

■ The ingredients of the proof.

Theorem **FUL−A**.

```
implies[subclass[x, FULL], full[A[x]]]
```

```
True
```

Theorem **ONISB−1B**

```
subclass[OMEGA, FULL]
```

```
True
```

Theorem **ISBELL−4**. Not in the **GOEDEL** program, but easily deduced:

```
Map[implies[#, member[x, OMEGA]] &,
    SubstTest[member, x, intersection[y, z], {y -> FULL, z -> P[OMEGA]}]     ] // Reverse
```

```
or[member[x, OMEGA], not[member[x, V]],
  not[subclass[x, OMEGA]], not[subclass[U[x], x]]] == True
```

```
or[member[x_, OMEGA], not[member[x_, V]],
  not[subclass[x_, OMEGA]], not[subclass[U[x_], x_]]] := True
```

Theorem **ISBELL−5**

```
implies[member[x, OMEGA], subclass[x, OMEGA]]
```

```
True
```

■ first steps

The following is just a restatement of Theorem **ONISB−1B**with a variable.

```
SubstTest[implies, and[subclass[x, y], subclass[y, z]], subclass[x, z],
    {y -> OMEGA, z -> FULL}]
```

```
or[not[subclass[x, OMEGA]], subclass[x, FULL]] == True
```

```
or[not[subclass[x_, OMEGA]], subclass[x_, FULL]] := True
```

From Theorem **FUL−A** we deduce:

```
Map[not, SubstTest[and, implies[p1, p2], implies[p2, p3], not[implies[p1, p3]],
        {p1 -> subclass[x, OMEGA], p2 -> subclass[x, FULL], p3 -> full[A[x]]}]]
```

```
or[not[subclass[x, OMEGA]], subclass[U[A[x]], A[x]]] == True
```

```
or[not[subclass[x_, OMEGA]], subclass[U[A[x_]], A[x_]]] := True
```

The following Corollary of **ISBELL−5**is used below.

```
SubstTest[implies, and[subclass[x, y], subclass[y, z]], subclass[x, z],
    {y -> OMEGA, z -> P[OMEGA]}]
```

```
or[not[subclass[x, OMEGA]], subclass[U[x], OMEGA]] == True
```

```
or[not[subclass[x_, OMEGA]], subclass[U[x_], OMEGA]] := True
```

■ Avoiding Skolem functions

The following strategem avoids the Skolem function **notsub[$c1,0]** that occurs in the **Otter** proof. This Skolem function is related to the variable **y** below.

```
Map[not, SubstTest[and, implies[p1, p3],
  implies[p2, p4], implies[and[p1, p2], p5], implies[p5, p6],
    implies[and[p3, p6], p7], not[implies[and[p1, p2], p7]],
        {p1 -> member[y, x], p2 -> subclass[x, OMEGA], p3 -> subclass[A[x], y],
        p4 -> subclass[x, P[OMEGA]], p5 -> member[y, OMEGA], p6 -> subclass[y, OMEGA],
        p7 -> subclass[A[x], OMEGA]}]]
```

```
or[not[member[y, x]], not[subclass[x, OMEGA]], subclass[A[x], OMEGA]] == True
```

The variable **y** is now eliminated:

```
Map[assert[forall[y, #]] &, %]
```

```
or[equal[0, x], not[subclass[x, OMEGA]], subclass[A[x], OMEGA]] == True
```

```
or[equal[0, x_], not[subclass[x_, OMEGA]], subclass[A[x_], OMEGA]] := True
```

■ Final steps

The following is just a special case of **ISBELL−4**.

```
SubstTest[implies, and[member[z, P[OMEGA]], member[z, FULL]], member[z, OMEGA], z -> A[x]]
```

```
or[equal[0, x], member[A[x], OMEGA],
  not[subclass[A[x], OMEGA]], not[subclass[U[A[x]], A[x]]]] == True
```

```
or[equal[0, x_], member[A[x_], OMEGA],
  not[subclass[A[x_], OMEGA]], not[subclass[U[A[x_]], A[x_]]]] := True
```

The final step is:

```
Map[not,
 SubstTest[and, implies[and[p1, p2], p3], implies[p1, p4], implies[and[p3, p4], p5],
    implies[and[p4, p6], p7], implies[and[p5, p7], p8], not[implies[and[p1, p2], p8]],
                 {p1 -> not[equal[0, x]],
   p2 -> subclass[x, OMEGA], p3 -> subclass[A[x], OMEGA],
        p4 -> member[A[x], V], p5 -> member[A[x], P[OMEGA]],
   p6 -> full[A[x]], p7 -> member[A[x], FULL],
        p8 -> member[A[x], OMEGA]}]]
```

```
or[equal[0, x], member[A[x], OMEGA], not[subclass[x, OMEGA]]] == True
```

This is Theorem **ON−A−1**.

```
or[equal[0, x_], member[A[x_], OMEGA], not[subclass[x_, OMEGA]]] := True
```