a counterexample related to FULL

Johan G. F. Belinfante
2006 January 8

In[1]:= **SetDirectory["l:"]; << goedel77.07a; << tools.m**

```
:Package Title: goedel77.07a                    2006 January 7 at 1:40 p.m.

It is now:  2006 Jan 8 at 4:51

Loading Simplification Rules

TOOLS.M                       Revised 2006 January 2

weightlimit = 40
```

summary

A counterexample is used to show that the class of full sets does not contain its own power class.

the ensemble tool

The **ensemble** tool in the file **tools.m** is useful for generating finite counterexamples.

In[3]:= **Begin["Goedel`Private`"];**

In[5]:= **?? bits**

```
bits[n] is a list of the bits of n in reversed order

bits[0] := {}

bits[n_] := Join[{Mod[n, 2]}, bits[Quotient[n, 2]]]
```

In[4]:= **?? ens**

```
ens[n] is the n-th set in the cumulative hierarchy (for finite n)

ens[0] := 0

ens[n_] := union @@ (set[ens[#1 - 1]] &) /@ Flatten[Position[bits[n], 1]]
```

counterexample

Cantor's theorem implies that **P[x]** cannot be a subclass of **x** when **x** is a set. This does not generalize to proper classes. In particular, the universal class **V**, the Russell class **RUSSELL** and the class **REGULAR** of all regular sets all have this property.

```
In[6]:= Map[subclass[P[#], #] &, {V, RUSSELL, REGULAR}]

Out[6]= {True, True, True}
```

The class **FULL** of all full sets is a proper class.

```
In[8]:= full[x]

Out[8]= subclass[U[x], x]

In[9]:= class[x, full[x]]

Out[9]= FULL

In[10]:= member[FULL, V]

Out[10]= False
```

In this section a counterexample is used to show that **P[FULL]** is not a subclass of **FULL**. This counterexample could be discovered using the **ensemble** tool as follows:

```
In[7]:= Select[Map[ens, Range[5]], member[#, dif[P[FULL], FULL]] &]

Out[7]= {set[set[0]]}
```

With this counterexample in hand, one readily establishes that the power class of **FULL** is not a subclass of **FULL**.

```
In[11]:= SubstTest[and, member[x, y], empty[y], {x → set[set[0]], y → dif[P[FULL], FULL]}]

Out[11]= subclass[P[FULL], FULL] == False

In[12]:= subclass[P[FULL], FULL] := False
```