range[SINGLETON as a partition

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```
In[1]:= SetDirectory["l:"]; << goedel86.10a; << tools.m

        :Package Title: goedel86.10a          2006 October 10 at 1:50 p.m.

        It is now:  2006 Oct 10 at 22:21

        Loading Simplification Rules

        TOOLS.M                      Revised 2006 October 10

        weightlimit = 40
```

summary

The general theory of (thin) equivalence relations is illustrated by means of the simplest of all equivalence relations, namely, identity relations. Singletons are the equivalence classes of the global identity relation **Id**. The class **range[SINGLETON]** of all singletons is a partition of the universal class **V**. The function **SINGLETON** is the canonical mapping which takes each set to the equivalence class to which it belongs.

the class of singletons as a partition

Theorem. The class of singletons is pairwise disjoint.

```
In[2]:= subclass[cart[range[SINGLETON], range[SINGLETON]], union[DISJOINT, Id]] // AssertTest

Out[2]= subclass[cart[range[SINGLETON], range[SINGLETON]], union[DISJOINT, Id]] == True

In[3]:= subclass[cart[range[SINGLETON], range[SINGLETON]], union[DISJOINT, Id]] := True
```

The canonical mapping for **Id** is **SINGLETON**.

```
In[4]:= composite[id[range[SINGLETON]], E]

Out[4]= SINGLETON
```

The inverse rewrite rule is also in place.

```
In[5]:= composite[inverse[E], id[range[SINGLETON]]]

Out[5]= inverse[SINGLETON]
```

The identity relation is the union of the class of cartesian squares of its equivalence classes.

```
In[6]:= composite[inverse[E], id[range[SINGLETON]], E]
Out[6]= Id
```

generalization

Theorem. The canonical map corresponding to the identity relation on any class **x** is a restriction of **SINGLETON**.

```
In[7]:= composite[id[image[SINGLETON, x]], E] // VSNormality
Out[7]= composite[id[image[SINGLETON, x]], E] == composite[SINGLETON, id[x]]

In[8]:= composite[id[image[SINGLETON, x_]], E] := composite[SINGLETON, id[x]]
```

Corollary.

```
In[9]:= composite[inverse[E], id[image[SINGLETON, x]]] // DoubleInverse
Out[9]= composite[inverse[E], id[image[SINGLETON, x]]] == composite[id[x], inverse[SINGLETON]]

In[10]:= composite[inverse[E], id[image[SINGLETON, x_]]] := composite[id[x], inverse[SINGLETON]]
```

The union of the cartesian squares of the equivalence classes is the equivalence relation **id[x]**.

```
In[11]:= composite[inverse[E], id[image[SINGLETON, x]], E]
Out[11]= id[x]
```